JENN

Training and Consultancy

The path to enlightened education

SUBJECT: SUBJECT NAME

GRADE 12

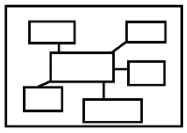
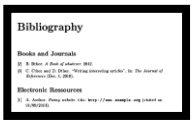
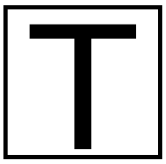
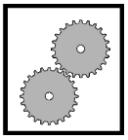
2025 LAST PUSH

SOLUTIONS MANUAL

TOPICS

1. ALL PAPER 1 TOPICS
2. ALL PAPER 2 TOPICS

ICON DESCRIPTION

 MIND MAP	 EXAMINATION GUIDELINE	 CONTENTS	 ACTIVITIES
 BIBLIOGRAPHY	 TERMINOLOGY	 WORKED EXAMPLES	 STEPS

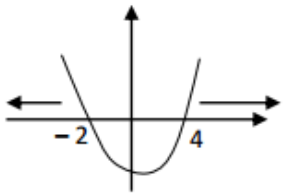
CONTENTS **PAGE**

Paper 1

Algebra, Equations and Inequalities

May/June 2024

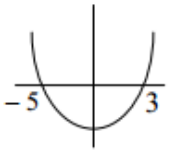
QUESTION 1/*VRAAG* 1

1.1.1	$3x^2+5x=0$ $x(3x+5)=0$ $x=0$ or $x=-\frac{5}{3}$	✓ $x=0$ ✓ $x=-\frac{5}{3}$ (2)
1.1.2	$4x^2+3x-5=0$ $x=\frac{-(3)\pm\sqrt{(3)^2-4(4)(-5)}}{2(4)}$ $x=0{,}80$ or $x=-1{,}55$	✓ correct substitution into correct formula ✓ answer ✓ answer (3)
1.1.3	$(x-1)^2-9\geq 0$ $x^2-2x-8\geq 0$ $(x-4)(x+2)\geq 0$ $x=4$ or $x=-2$ $x\leq -2$ or $x\geq 4$ 	✓ standard form ✓ critical values ✓✓ $x\leq -2$ or $x\geq 4$ (4)
1.1.4	$5^{2x}-5^x=0$ $5^x(5^x-1)=0$ $5^x\neq 0$ or $5^x=1$ $x=0$ **OR/OF** $5^{2x}=5^x$ $2x=x$ $2x-x=0$ $x=0$	✓ common factor ✓ $5^x\neq 0$ ✓ $5^x=1$ ✓ $x=0$ (4) **OR/OF** ✓ $5^{2x}=5^x$ ✓ $2x=x$ ✓ $2x-x=0$ ✓ $x=0$ (4)

1.1.5	$\frac{x}{\sqrt{20-x}}=1$ $x=\sqrt{20-x}$ $x^2=20-x$ $x^2+x-20=0$ $(x+5)(x-4)=0$ $x=4$ or $x\neq -5$	✓ isolating the surd ✓ squaring both sides ✓ standard form ✓ answers ✓ selection (5)
1.2	$2x^2-y^2=7$ .... (1) $x+y=9$ .... (2) $y=9-x$ $2x^2-(9-x)^2=7$ $2x^2-81+18x-x^2=7$ $x^2+18x-88=0$ $(x+22)(x-4)=0$ $x=-22$ or $x=4$ $y=31$ or $y=5$ **OR/*OF*** $2x^2-y^2=7$ .... (1) $x+y=9$ .... (2) $x=9-y$ $2(9-y)^2-y^2=7$ $2(81-18y+y^2)-y^2-7=0$ $162-36y+2y^2-y^2-7=0$ $y^2-36y+155=0$ $(y-31)(y-5)=0$ $y=31$ or $y=5$ $x=-22$ or $x=4$	✓ $y=9-x$ ✓ substitution ✓ standard form ✓ x-values ✓ y-values (5) **OR/*OF*** ✓ $x=9-y$ ✓ substitution ✓ standard form ✓ y-values ✓ x-values (5)
1.3	$P\times T=(1-a)(1+a)(1+a^2)(1+a^4)..(1+a^{512})$ $P\times T=(1-a^2)(1+a^2)(1+a^4)...(1+a^{512})$ $P\times T=(1-a^4)(1+a^4)..(1+a^{512})$ $P\times T=(1-a^8)..(1+a^{512})$ $P\times T=(1-a^{512})(1+a^{512})$ $=1-a^{1024}$	✓ $(1-a^4)$ ✓ $(1-a^{512})$ ✓ $1-a^{1024}$ (3)
		[26]

May/June 2023

QUESTION 1/*VRAAG* 1

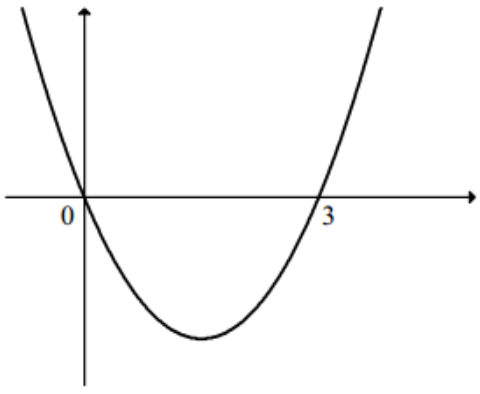
1.1.1	$x^2-7x+12=0$ $(x-4)(x-3)=0$ $x=4$ or $x=3$ Answer only: Full Marks	✓ factors ✓ $x=4$ ✓ $x=3$ (3)
1.1.2	$3x^2+5x-1=0$ $x=\frac{-5\pm\sqrt{5^2-4(3)(-1)}}{2(3)}=\frac{-5\pm\sqrt{37}}{6}$ $\therefore x=0{,}18$ or $x=-1{,}85$	✓standard form ✓substitution into the correct formula ✓ $x=0{,}18$ ✓ $x=-1{,}85$ (4)
1.1.3	$x^2+2x-15<0$ $(x-3)(x+5)<0$ $x=3$ or $x=-5$ $-5<x<3$ 	✓standard form ✓ critical values ✓✓answer (4)
1.1.4	$\sqrt{2(1-x)}=x-1$ $\left(\sqrt{2(1-x)}\right)^2=(x-1)^2$ $2-2x=x^2-2x+1$ $x^2-1=0$ $\therefore x=1$ and $x\neq-1$	✓squaring both sides ✓simplification ✓standard form ✓ answer with selection (4)

1.2	$3^{x+y}=27$	
	$x^2+y^2=17$	
	$3^{x+y}=3^3$	✓ $3^{x+y}=3^3$
	$x+y=3$ .....(1)	✓ $x+y=3$
	$y=3-x$	
	$x^2+(3-x)^2=17$	✓substitution
	$2x^2-6x-8=0$	
	$x^2-3x-4=0$	✓standard form
	$(x-4)(x+1)=0$	
	$x=4$ or $x=-1$	✓x-values
	$y=-1$ or $y=4$	✓y-values (6)
	OR/*OF*	**OR/*OF***
	$3^{x+y}=27$	
	$x^2+y^2=17$	
	$3^{x+y}=3^3$	✓ $3^{x+y}=3^3$
	$x+y=3$ .....(1)	✓ $x+y=3$
	$x=3-y$	
	$(3-y)^2+y^2=17$	✓substitution
	$9-6y+y^2+y^2-17=0$	
	$2y^2-6y-8=0$	✓standard form
	$y^2-3y-4=0$	
	$(y-4)(y+1)=0$	
	$y=-1$ or $y=4$	✓y-values
	$x=4$ or $x=-1$	✓x-values (6)

1.3	$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{99}+\sqrt{100}}$ $=\frac{1}{\sqrt{1}+\sqrt{2}}\times\frac{\sqrt{1}-\sqrt{2}}{\sqrt{1}-\sqrt{2}}+\ldots$ $+\frac{1}{\sqrt{99}+\sqrt{100}}\times\frac{\sqrt{99}-\sqrt{100}}{\sqrt{99}-\sqrt{100}}$ $=-1+\sqrt{2}-\sqrt{2}+\sqrt{3}-\sqrt{3}+2\ldots-\sqrt{99}+10$ $=-1+10$ $=9$	✓rationalisation ✓ simplification ✓ answer (3)
		[24]

May/June 2022

QUESTION/*VRAAG* 1

1.1.1	$x^2+2x-15=0$ $(x+5)(x-3)=0$ $x=-5$ or $x=3$	✓ factors ✓ $x=-5$ ✓ $x=3$ (3)
1.1.2	$5x^2-x-9=0$ $x=\frac{-(-1)\pm\sqrt{(-1)^2-4(5)(-9)}}{2(5)}$ $x=\frac{1\pm\sqrt{181}}{10}$ $x=1{,}45$ or $x=-1{,}25$	✓substitution into the correct formula ✓ $x=1{,}45$ ✓ $x=-1{,}25$ (3)
1.1.3	$x^2\leq 3x$ $x^2-3x\leq 0$ $x(x-3)\leq 0$  $0\leq x\leq 3$ OR $x\in[0;3]$	✓ standard form ✓ factors ✓✓ answer (4)

1.2.1	$a+\frac{64}{a}=16$ $a^2-16a+64=0$ $(a-8)^2=0$ $a=8$	✓ standard form ✓ factors ✓ answer (3)
1.2.2	$2^x+2^{6-x}=16$ $2^x+\frac{64}{2^x}=16$ $2^x=8$ (from 1.2.1) $2^x=2^3$ $x=3$	 ✓ exp law ✓ $2^x=8$ ✓ answer (3)
1.3	$\sqrt{\frac{2^{1002}(1+2^4)}{17(2)^{998}}}$ $=\sqrt{\frac{2^4(17)}{17}}$ $=\sqrt{2^4}$ $=2^2$ $=4$	✓common factor ✓second factor ✓simplification ✓answer (4)

1.4 $2x-y=2 \quad \ldots(1)$

$\frac{1}{x}-3y=1 \quad \ldots(2)$

$y=2x-2$ ✓ $y=2x-2$

$\frac{1}{x}-3(2x-2)=1$ ✓substitution

$\frac{1}{x}-6x+6-1=0$ ✓simplification

$1-6x^2+6x-x=0$

$-6x^2+5x+1=0$

$6x^2-5x-1=0$ ✓standard form

$(6x+1)(x-1)=0$

$x=-\frac{1}{6} \quad \text{or} \quad x=1$ ✓x-values

$y=2\left(-\frac{1}{6}\right)-2 \quad \text{or} \quad y=2(1)-2$

$y=-\frac{7}{3} \quad \text{or} \quad y=0$ ✓y-values

(6)

OR/*OF*

$x = \frac{2+y}{2} \quad \ldots(1)$

$\frac{1}{x} - 3y = 1 \quad \ldots(2)$

$\frac{1}{\frac{2+y}{2}} - 3y = 1$

$\frac{2}{2+y} - 3y = 1$

$\frac{2-6y-3y^2}{2+y} = 1$

$2 - 6y - 3y^2 = 2 + y$

$-3y^2 - 7y = 0$

$-y(3y+7) = 0$

$y = 0 \quad \text{or} \quad y = -\frac{7}{3}$

$x = 1 \quad \text{or} \quad x = -\frac{1}{6}$

OR/*OF*

✓ $x = \frac{2+y}{2}$

✓substitution

✓simplification

✓standard form

✓*y*-values

✓*x*-values

(6)
[26]

Patterns and Sequences

May/June 2024

QUESTION 2/*VRAAG* 2

2.1.1	$r=\frac{1}{2}$ Yes, because $-1<\frac{1}{2}<1$	✓ $r=\frac{1}{2}$ ✓ answer with reason (2)
2.1.2	$S_\infty=\frac{a}{1-r}$ $S_\infty=\frac{4}{1-\frac{1}{2}}$ $\therefore S_\infty=8$	✓ substitution ✓ answer (2)
2.2	$\sum_{p=k}^{10}3^{p-1}=3^{k-1}+3^{k+1-1}+3^{k+2-1}+\ldots+3^9$ $=3^{k-1}+3^k+3^{k+1}+\ldots 3^9$ $S_n=\frac{a\left(r^n-1\right)}{r-1}$ $29\,520=\frac{3^{k-1}\left(3^{11-k}-1\right)}{3-1}$ $3^{10}-3^{k-1}=59\,040$ $3^{k-1}=9$ $k-1=2$ $\therefore k=3$	✓ $a=3^{k-1}$ ✓ $r=3$ ✓ $n=11-k$ ✓ substitution ✓ answer (5)
		[9]

QUESTION 3/*VRAAG* 3

3.1.1	$3\ ;\ 7\ ;\ 12\ ;\ 18$ $\vee\ \ \vee\ \ \vee$ First diff: $4\ ;\ 5\ \ ;\ 6$ $\vee\ \ \vee$ Second diff: $1\ \ ;\ 1$ $2a=1$ $a=\frac{1}{2}$ $3a+b=4$ $3\left(\frac{1}{2}\right)+b=4$ $b=\frac{5}{2}$ $a+b+c=3$ $\frac{1}{2}+\frac{5}{2}+c=3$ $c=0$ $T_n=\frac{1}{2}n^2+\frac{5}{2}n$	✓ $2a=1$ ✓ $3\left(\frac{1}{2}\right)+b=4$ ✓ $\frac{1}{2}+\frac{5}{2}+c=3$ (3)
3.1.2	$13\,527=\frac{1}{2}n^2+\frac{5}{2}n$ $n^2+5n-27\,054=0$ $(n-162)(n+167)=0$ $n=162$ or $n=-167$ $T_{161}=13\,363$ $\therefore T_{161}+164=13\,527$ 164 must be added. **OR/*OF*** $T_n=3+$ sum of 1st differences $13527=3+4+5+\ldots+n$ $\frac{n-3+1}{2}[3+n]=13527$ $n^2+n-27060=0$ $(n+165)(n-167)=0$ $n=164$	✓ $13\,527=\frac{1}{2}n^2+\frac{5}{2}n$ ✓ standard form ✓ answers for n ✓ 164 (4) **OR/OF** ✓ $13527=3+4+5+\ldots+n$ ✓ $n^2+n-27060=0$ ✓ answers for n ✓ 164 (4)

3.2.1	$T_n = 8+(n-1)(3)$ $T_n = 3n+5$ $41 = 3n+5$ $36 = 3n$ $n = 12$	✓ $T_n = 3n+5$ ✓ $T_n = 41$ ✓ answer (3)
3.2.2a	$P_{41} = 12$	✓ answer (1)
3.2.2b	$P_8 = a+7d = 1$ $P_{11} = a+10d = 2$ $3d = 1$ $d = \frac{1}{3}$ $a+7\left(\frac{1}{3}\right) = 1$ $a = -\frac{4}{3}$	✓ $a+7d = 1$ ✓ $a+10d = 2$ ✓ value of d ✓ value of a (4)
		[15]

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QUESTION 2/*VRAAG* 2

2.1.1	$\frac{1}{5}+\frac{1}{15}+\frac{1}{45}+\ldots$ $r=\frac{\frac{1}{15}}{\frac{1}{5}}=\frac{1}{3}$ $-1<\frac{1}{3}<1$ $\therefore$ the series is convergent.	✓ $r=\frac{1}{3}$ ✓ answer (any indicator of convergence) (2)
2.1.2	$S_\infty=\frac{a}{1-r}$ $=\frac{\frac{1}{5}}{1-\frac{1}{3}}$ $=\frac{3}{10}$	✓ substitution ✓ answer (2)
2.2.1	$4x\ ;\ \frac{1}{81}$	✓ $4x$ ✓ $\frac{1}{81}$ (2)
2.2.2	$T_n=x+(n-1)x$ $=x+xn-x$ $=xn$ Answer only: Full Marks	✓ substitution ✓ answer (2)
2.2.3	$T_n=ar^{n-1}$ $T_{13}=\frac{1}{3}\left(\frac{1}{3}\right)^{13-1}$ $T_{13}=\left(\frac{1}{3}\right)^{13}$ or $\frac{1}{1\,594\,323}$ or $6{,}27\times10^{-7}$ or 3^{-13}	✓ $n=13$ ✓ $r=\frac{1}{3}$ ✓ answer (3)
2.2.4	$\sum_{n=1}^{21}P_n=S_{11}+S_{10}$ $=\frac{11}{2}[2x+10x]+\frac{\frac{1}{3}\left[1-\left(\frac{1}{3}\right)^{10}\right]}{1-\frac{1}{3}}$ $=66x+0{,}5$ $33{,}5=66x+0{,}5$ $\therefore x=\frac{1}{2}$	✓ S_{11} ✓ $+S_{10}$ ✓ arithmetic sum ✓ geometric sum ✓ $66x+0{,}5$ (A) ✓ answer (6)
		[17]

QUESTION 3/*VRAAG* 3

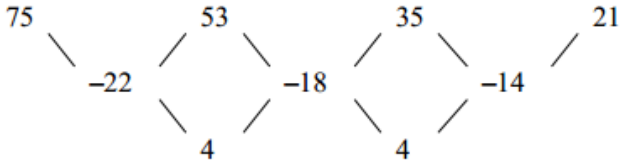
3.1	$x \;;\; x \;;\; T_3 \;;\; \ldots$ $0 \quad T_3 - x$ 10	
	$2a = 10 \qquad 3a + b = 0$ $a = 5 \qquad\quad b = -15$	✓ $2a = 10$ ✓ $3a + b = 0$
	$T_3 - x - 0 = 10$ $\therefore T_3 = x + 10$	✓ $T_3 = x + 10$
	$2x + T_3 = 28$ $2x + x + 10 = 28$ $3x = 18$ $x = 6$	✓ $2x + T_3 = 28$ ✓ $x = 6$
	$a + b + c = 6$ $5 - 15 + c = 6$ $c = 16$	✓ $5 - 15 + c = 6$ (6)
	$\therefore T_n = 5n^2 - 15n + 16$	
	OR/*OF*	**OR/*OF***
	$2a = 10$ $\therefore a = 5$	✓ $2a = 10$
	$T_1 = a + b + c \quad T_2 = 4a + 2b + c \quad T_3 = 9a + 3b + c$ $= 5 + b + c \quad = 20 + 2b + c \quad = 45 + 3b + c$	
	$5 + b + c = 20 + 2b + c$ $b = -15$	✓ $5 + b + c = 20 + 2b + c$
	$T_1 = -10 + c \quad T_2 = -10 + c \quad T_3 = c$	✓ $T_1 = -10 + c$ ✓ $T_2 = -10 + c$
	$T_1 + T_2 + T_3 = -10 + c - 10 + c + c$ $28 = 3c - 20$ $c = 16$	✓ $28 = 3c - 20$ ✓ $c = 16$ (6)

3.2	$T_n = 5n^2 - 15n + 16$ $216 = 5n^2 - 15n + 16$ $5n^2 - 15n - 200 = 0$ $n^2 - 3n - 40 = 0$ $(n-8)(n+5) = 0$ $n = 8 \quad \text{or} \quad n \neq -5$ $\therefore T_8 = 216$	✓ equating ✓ standard form ✓ $n = 8$ (3)
		[9]

May/June 2022

QUESTION/*VRAAG* 2

2.1.1	$a + 6d = 35$ $-1 + 6d = 35$ $6d = 36$ $d = 6$ **OR/OF** $\frac{35-(-1)}{7-1} = 6$ ANSWER ONLY: FULL MARKS	✓ substitution ✓answer (2) **OR/OF** ✓ substitution ✓answer (2)
2.1.2	$T_n = a + (n-1)d$ $473 = -1 + (n-1)(6)$ $79 = n - 1$ $\therefore n = 80$ ANSWER ONLY: FULL MARKS	✓substitution into the correct formula ✓ equating to 473 ✓ answer (3)
2.1.3	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{40} = \frac{40}{2}[2(-1) + (40-1)(6)]$ $\therefore S_{40} = 4640$ **OR/*OF*** $T_{40} = 6(40) - 7$ $= 233$ $S_n = \frac{n}{2}(a + l)$ $= \frac{40}{2}(-1 + 233)$ $= 4640$	✓ substitution ✓answer (2) **OR/*OF*** ✓ substitution ✓answer (2)

2.2.1	 $T_5 = 11$	✓answer (A) (1)
2.2.2	$T_n = an^2 + bn + c$ $2a = 4$ $a = 2$ $3a + b = -22$ $6 + b = -22$ $b = -28$ $a + b + c = 75$ $2 - 28 + c = 75$ $c = 101$ $\therefore T_n = 2n^2 - 28n + 101$	✓ $T_n = an^2 + bn + c$ ✓ $a = 2$ ✓ $b = -28$ ✓ $c = 101$ (4)

2.2.3

Minimum value of T_n

$$n = -\frac{b}{2a} = -\frac{(-28)}{2(2)}$$

$n = 7$ ✓ $n = 7$

Minimum value of $T_n = 2(7)^2 - 28(7) + 101 = 3$ ✓ min value = 3

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern. ✓ $-\frac{1}{5}$ value of term of old pattern

Maximum value of new pattern $= -\frac{3}{5}$ ✓ max value $= -\frac{3}{5}$ (4)

OR/*OF*

OR/*OF*

$T_n' = 4n - 28$

$4n - 28 = 0$

$4n = 28$

$n = 7$ ✓$n = 7$

Minimum value of $T_n = 2(7)^2 - 28(7) + 101 = 3$ ✓ min value = 3

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern. ✓ $-\frac{1}{5}$ value of term of old pattern

Maximum value of new pattern $= -\frac{3}{5}$ ✓ max value $= -\frac{3}{5}$

(4)

OR/*OF*

$T_n' = 4n - 28$

$4n - 28 = 0$

$4n = 28$

$n = 7$

Minimum value of $T_n = 2(7)^2 - 28(7) + 101 = 3$

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern.

Maximum value of new pattern $= -\frac{3}{5}$

OR/OF

$T_n = -\frac{2}{5}n^2 + \frac{28}{5}n - \frac{101}{5}$

$n = -\frac{b}{2a} = \frac{-\frac{28}{5}}{2\left(\frac{-2}{5}\right)}$

$= 7$

$T_7 = -\frac{3}{5}$

OR/OF

OR/*OF*

✓$n = 7$

✓ min value = 3

✓ $-\frac{1}{5}$ value of term of old pattern

✓ max value $= -\frac{3}{5}$

(4)

OR/OF

✓✓ $T_n \div (-5)$

✓$n = 7$

✓ max value $= -\frac{3}{5}$ (4)

OR/OF

$$T_n = -\frac{2}{5}n^2 + \frac{28}{5}n - \frac{101}{5}$$

$$T'_n = -\frac{4}{5}n + \frac{28}{5}$$ ✓✓ $T_n \div (-5)$

$$-\frac{4}{5}n + \frac{28}{5} = 0$$

$$-4n = -28$$

$$n = 7$$

Minimum value of $T_n = 2(7)^2 - 28(7) + 101 = 3$ ✓$n = 7$

Each term in the new pattern is $-\frac{1}{5}$ the value of the terms in the old pattern.

Maximum value of new pattern $= -\frac{3}{5}$

✓ max value $= -\frac{3}{5}$

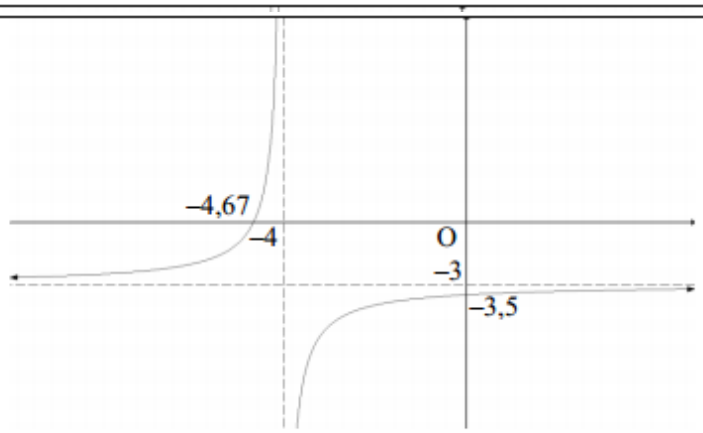
(4)

[16]

Functions and Graphs

May/June 2021

QUESTION/*VRAAG* 4

4.1	$x+1=-x-7$ $2x=-8$ $x=-4$ $\therefore y=-3$ $\therefore f(x)=\frac{-2}{x+4}-3$ $\therefore p=4$ and $q=-3$ **OR/OF** $p+q=1$ .....(1) $-p+q=-7$ $q=p-7$ .....(2) subs. (2) into (1) $p+p-7=1$ $2p=8$ $p=4$ $q=-3$	✓ $x+1=-x-7$ ✓ $2x=-8$ ✓ $x=-4$ ✓ $y=-3$ (4) **OR/OF** ✓ $p+q=1$ ✓ $q=p-7$ ✓ substitution ✓ simplification (4)
4.2	$y=\frac{-2}{x+4}-3$ $0=\frac{-2}{x+4}-3$ $-2-3(x+4)=0$ $-3x-14=0$ $\therefore x=-\frac{14}{3}$	✓ $y=0$ ✓ $x=-\frac{14}{3}$ (2)
4.3		✓ horizontal asymptote ✓ vertical asymptote ✓ y intercept ✓ shape (4)
		[10]

QUESTION/*VRAAG* 5

5.1	$-2x^2+4x+16=0$ $x^2-2x-8=0$ $(x-4)(x+2)=0$ $x=4$ or $x=-2$ $\therefore$ A$(-2;0)$ and B$(4;0)$	✓ factors ✓ $x=-2$ ✓ $x=4$ (3)
5.2	$f(x)=-2x^2+4x+16$ $-\frac{b}{2a}=-\frac{-4}{-2(2)}=1$ $f(1)=-2(1)^2+4(1)+16=18$ $\therefore$ C$(1;18)$ **OR/OF** $f(x)=-2x^2+4x+16$ $f'(x)=-4x+4$ $-4x+4=0$ $x=1$ $f(1)=-2(1)^2+4(1)+16=18$ $\therefore$ C$(1;18)$	✓ 1 ✓ 18 (2) **OR/OF** ✓ 1 ✓ 18 (2)
5.3	$y \leq 18$ **OR/OF** $y\in(-\infty;18]$	✓ $y \leq 18$ (1) **OR/OF** ✓ $y\in(-\infty;18]$ (1)
5.4	TP (1 ; 18) for f TP (2 ; 15) for h $\therefore p=-1 \quad q=-3$	✓ TP for h at (2 ; 15) ✓ $p=-1$ ✓ $q=-3$ (3)
5.5	$y=2x+4$ $x=2y+4$ $\therefore y=\frac{1}{2}x-2$	✓swop x and y ✓ $y=\frac{1}{2}x-2$ (2)
5.6	$g(x)=0$ or $g^{-1}(x)=0$ $x=4$ or $x=-2$ (product 0 at x-intercepts)	✓ $x=4$ ✓ $x=-2$ (2)

5.7	$-2x^2+4x+16+k=2x+4$ $-2x^2+2x+12+k=0$ $b^2-4ac<0$ $(2)^2-4(-2)(12+k)<0$ $4+8(12+k)<0$ $100+8k<0$ $k<-12{,}5$ **OR/OF** $g'(x)=2$ $f'(x)=-4x+4=2$ $x=\frac{1}{2}$ $f\left(\frac{1}{2}\right)=17{,}5$ $g\left(\frac{1}{2}\right)=5$ $\therefore k<-12{,}5$	✓equating ✓ standard form ✓ $b^2-4ac<0$ ✓ substitution ✓ answer (5) **OR/OF** ✓ $g'(x)=2$ ✓ $f'(x)=-4x+4$ ✓ $f\left(\frac{1}{2}\right)=17{,}5$ ✓ $g\left(\frac{1}{2}\right)=5$ ✓ answer (5)
		[18]

QUESTION/*VRAAG* 6

6.1.1	$y=3^x$ $x=3^y$ $y=\log_3 x$	✓ swop x and y ✓ equation (2)
6.1.2	$h(x)=3^{x-4}+2$ Transformation: 4 units left, 2 units down $P'(2;9)$	✓ $x=2$ (A) ✓ $y=9$ (A) (2)
6.2	$f(x)=2^{x+p}+q$ $q=-16$ $16=2^{p+3}-16$ $2^{p+3}=32$ $2^{p+3}=2^5$ $\therefore p+3=5$ $p=2$	✓ $q=-16$ ✓ substitute (3 ; 16) ✓ $2^{p+3}=2^5$ or $p+3=\log_2 32$ ✓ $p=2$ (4)
		[8]

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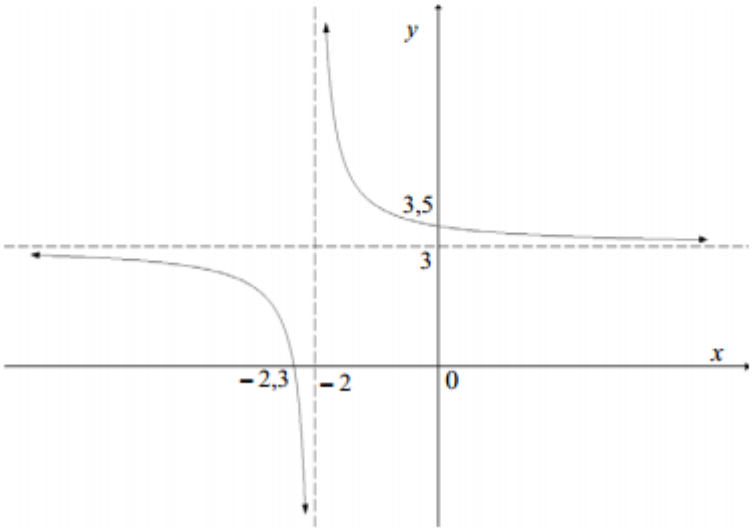
QUESTION/*VRAAG* 4

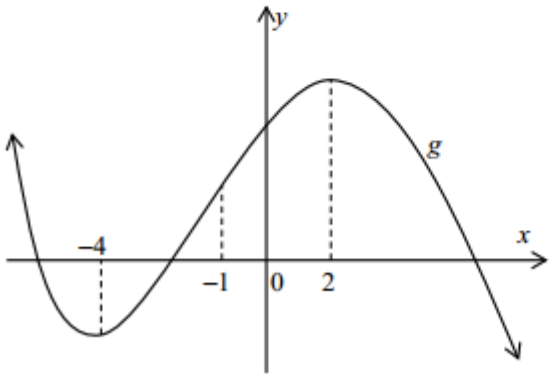
4.1	$y > 0$ **OR/*OF*** $y \in (0\ ;\ \infty)$	✓answer (1) **OR/*OF*** ✓answer (1)
4.2	$g: y = \left(\frac{1}{2}\right)^x$ $g^{-1}: x = \left(\frac{1}{2}\right)^y$ $y = \log_{\frac{1}{2}} x$ or $y = -\log_2 x$ or $y = \log_2 \frac{1}{x}$	 ✓ $x = \left(\frac{1}{2}\right)^y$ ✓ equation (2)
4.3	Yes. The vertical line test cuts g^{-1} once *Ja. Die vertikale lyn toets sny g^{-1} slegs eenkeer.* **OR/*OF*** Yes. For every x-value there is a unique y-value *Ja. Vir elke x-waarde is daar 'n unieke y-waarde* **OR/*OF*** Yes. g is a one-to-one function / *Ja. g is 'n een-tot-een funksie* **OR/*OF*** Yes. The horizontal line cuts g only once *Ja. Die horisontale lyn sny g slegs een keer*	✓ yes ✓ valid reason (2) **OR/*OF*** ✓ yes ✓ valid reason (2) **OR/*OF*** ✓ yes ✓ valid reason (2) **OR/*OF*** ✓ yes ✓ valid reason (2)

4.4.1	$y = -\log_2 x$ $2 = -\log_2 a$ $a = 2^{-2} = \frac{1}{4}$ or $a = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$	✓ correct subst into correct formula $(a\ ;\ 2)$ ✓answer (2)
4.4.2	$M'\left(2;\frac{1}{4}\right)$ or $M'(2;a)$	✓ answer (1)
4.5	$M''\left(-1;\frac{9}{4}\right)$	✓ -1 ✓✓ $\frac{9}{4}$ (3)
		[11]

QUESTION/*VRAAG* 5

5.1.1	$x=-2$ $y=3$	✓ answer ✓ answer (2)
5.1.2	$\left(0;\frac{7}{2}\right)$	✓ answer (1)
5.1.3	$\frac{1}{x+2}+3=0$ $1+3(x+2)=0$ $3x=-7$ $x=-\frac{7}{3}$ x-intercept $\left(-\frac{7}{3};0\right)$	✓ $y=0$ ✓ answer (2)
5.1.4		✓asymptotes at $y=3$ and $x=-2$ ✓intercepts at $y=3,5$ and $x=-2,3$ ✓shape (reasonable representation in correct quadrants) (3)

5.2.1	$-2x+4=0$ $2x=4$ $x=2$ $\therefore$ S(2 ; 0)	✓ $y=0$ ✓ $x=2$ (2)
5.2.2	Equation of k: $y=a(x+1)^2+18$ $0=a(2+1)^2+18$ or $0=a(-4+1)^2+18$ $9a=-18$ $a=-2$ $y=-2(x+1)^2+18$	✓ $y=a(x+1)^2+18$ ✓ substitute (2 ; 0) or (−4 ; 0) ✓ a (3)

5.2.3	$-2x^2-4x+16=-2x+4$ $-2x^2-2x+12=0$ $x^2+x-6=0$ $(x+3)(x-2)=0$ $x=-3$ or $x=2$ $y=-2(-3)+4=10$ T(−3 ; 10)	✓ equating ✓ standard form ✓ factors ✓ choosing $x=-3$ ✓ answer (5)
5.2.4	$x<-3$ or $x>2$ **OR/*OF*** $(-\infty\ ;-3)\cup(2\ ;\infty)$	✓✓ answer (2) **OR/*OF*** ✓✓ answer (2)
5.2.5(a)	$x<-1$ **OR/*OF*** $(-\infty\ ;-1)$	✓✓ answer (2) **OR/*OF*** ✓✓ answer (2)
5.2.5(b)		✓ shape of cubic with local min tp moving to local max tp ✓ turning points at $x=2$ and $x=-4$ ✓ point of inflection at $x=-1$ (3) **[25]**

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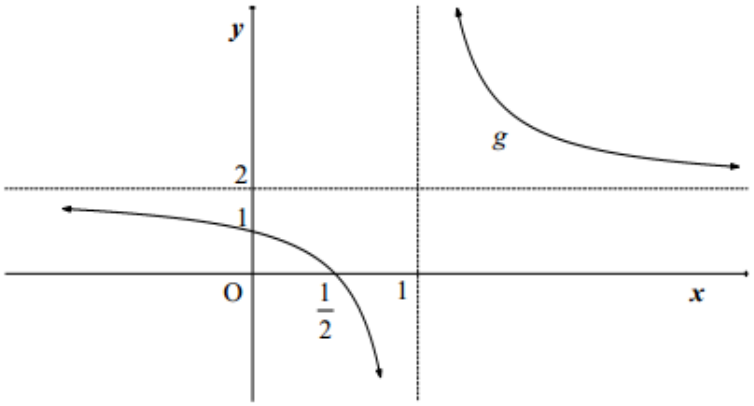
QUESTION 4/*VRAAG* 4

4.1	$x = 1$ $y = 2$	✓ $x = 1$ ✓ $y = 2$ (2)
4.2		✓ x-intercept ✓ y-intercept ✓ asymptotes ✓ shape (4)
4.3	$x < \frac{1}{2}$ or $x > 1$	✓ $x < \frac{1}{2}$ ✓ $x > 1$ (2)
4.4	$y = -(x-1)+2$ $y = -x+3$ **OR/*OF*** $y-2 = -(x-1)$ $y = -x+3$ **OR/OF** $y = -x+c$ $2 = -(1)+c$ $c = 3$ $\therefore y = -x+3$	✓ substitution of (1 ; 2) ✓ answer (2) **OR/*OF*** ✓ substitution of (1 ; 2) ✓ answer (2) **OR/OF** ✓ substitution of (1 ; 2) ✓ answer (2)
		[10]

QUESTION5/*VRAAG 5*

5.1	$P'(2;4)$	✓ $x=2$ ✓ $y=4$ (2)
5.2	$f(x)=\log_a x$ $2=\log_a 4$ $a^2=4$ $a=2$	✓substitute (4 ; 2) ✓ $a^2=4$ (2)
5.3	$y=2^x$	✓ $y=2^x$ (1)
5.4	$1=\log_2 x$ $\therefore x=2$ T(2 ; 1) RT = 2 units P'T = 3 units Area of $\Delta RTP' = \frac{1}{2}.RT.TP'$ $= \frac{1}{2}\times 2\times 3 = 3 \text{ units}^2$	✓ $x=2$ ✓ RT = 2 units ✓ P'T = 3 units ✓ answer (4)
		[9]

QUESTION 6/*VRAAG 6*

6.1	$y \geq -4$ or $y \in [-4;\infty)$	✓ answer (1)
6.2	$x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3$ or $x = -1$ $\therefore$ E(3 ; 0) and D(−1 ; 0)	✓= 0 ✓ both x-values ✓correct identification of coordinates (3)
6.3	P(0 ; − 3) $\therefore m_{PE} = 1$ $\therefore g(x) = x - 3$	✓ $m_{PE} = 1$ ✓ $g(x) = x - 3$ (2)
6.4	$f(x) > g(x)$ $x < 0$ or $x > 3$	✓ $x < 0$ ✓ $x > 3$ (2)
6.5	Distance $= -x^2 + 2x + 3 - x + 3 = -x^2 + x + 6$ $D' = -2x + 1 = 0$ or/of $x = -\frac{b}{2a}$ $\therefore x = \frac{1}{2}$ $\therefore x = \frac{1}{2}$ $D\left(\frac{1}{2}\right) = -\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 6$ $= \frac{25}{4} = 6{,}25$	✓ $D = -x^2 + x + 6$ ✓method ✓ $x = \frac{1}{2}$ ✓ substitution ✓ answer (5)
6.6	$f'(x) = m$ $1 = 2x - 2$ $x = \frac{3}{2}$ Point on f: $\left(\frac{3}{2}; \frac{-15}{4}\right)$ $-\frac{15}{4} = \left(\frac{3}{2} - 3\right) - n$ $\therefore n = 2\frac{1}{4} = \frac{9}{4} = 2{,}25$	✓ $1 = 2x - 2$ ✓ $x = \frac{3}{2}$ ✓ $\frac{-15}{4}$ ✓ $-\frac{15}{4} = \left(\frac{3}{2} - 3\right) - n$ ✓ answer (5)
	OR/OF $f(x) = k(x)$ $x^2 - 2x - 3 = x - 3 - n$ $x^2 - 3x + n = 0$ $\Delta = b^2 - 4ac$ $= (-3)^2 - 4(1)(n)$ To touch : $\Delta = 0$ $0 = 9 - 4n$ $4n = 9$ $n = \frac{9}{4}$	**OR/OF** ✓equating ✓ standard form ✓substitution into Δ ✓ $\Delta = 0$ ✓ answer (5)
		[18]

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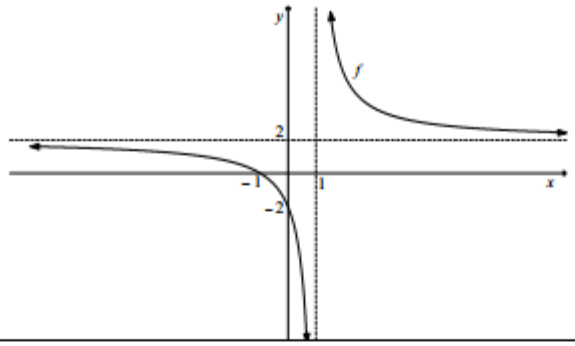
QUESTION 4/*VRAAG* 4

4.1.1	decreasing	✓ decreasing (1)
4.1.2	$y=\left(\frac{1}{3}\right)^x$ $x=\left(\frac{1}{3}\right)^y$ $\therefore y=\log_{\frac{1}{3}} x$ **OR/OF** $y=3^{-x}$ $x=3^{-y}$ $\therefore y=-\log_3 x$	✓ swop x and y ✓ answer (2) **OR/OF** ✓ swop x and y ✓ answer (2)
4.1.3	$x>0; x\in R$	✓ answer (1)
4.1.4	$y=-5$	✓ answer (1)
4.2.1	$x=1$ $y=2$	✓ $x=1$ ✓ $y=2$ (2)
4.2.2	$\frac{4}{x-1}+2=0$ $4=-2x+2$ $2x=-2$ $x=-1$	✓ let $y=0$ ✓ $x=-1$ (2)

4.2.3		✓ asymptotes ✓ x-intercept ✓ y-intercept ✓ shape (4)
4.2.4	$\frac{4}{x-1} \geq -2$ $\frac{4}{x-1} + 2 \geq 0$ $x \leq -1$ or $x > 1$	✓ $x \leq -1$ ✓ $x > 1$ (2)
4.2.5	$y = -x + c$ $2 = -3 + c$ $c = 5$ $y = -x + 5$ **OR/*OF*** $y = -x + c$ $2 = -1 + c$ $c = 3$ $y = -x + 3$ $y = -(x-2) + 3$ $y = -x + 5$ **OR/OF** $y = -(x+p) + q$ $y = -((x-2) + (-1)) + 2$ $y = -x + 5$	✓ intersection of axes at (3 ; 2) ✓ subst (3 ; 2) and $m = -1$ ✓ $y = -x + 5$ (3) **OR/*OF*** ✓✓ $-(x-2)+3$ ✓ $y = -x + 5$ (3) **OR/*OF*** ✓✓ $y = -((x-2) + (-1)) + 2$ ✓ $y = -x + 5$ (3)
		[18]

QUESTION 5/*VRAAG* 5

5.1	T.P$(-3;4)$	✓-3 ✓ 4 (2)
5.2	$y \le 4$ or $y \in (-\infty;4]$	✓answer (1)
5.3	$f(x) = g(x)$ $-(x+3)^2 + 4 = x+5$ $-x^2 - 6x - 9 + 4 = x+5$ $-x^2 - 7x - 10 = 0$ $x^2 + 7x + 10 = 0$ $(x+5)(x+2) = 0$ $x = -5$ or $x = -2$	✓ equating ✓ $-x^2 - 6x - 9$ ✓ standard form ✓ factors (4)
5.4	The graph must shift more than 2 and less than 5 units to the right $\therefore -5 < c < -2$	✓✓answer (2)
5.5	$D(x) = f(x) - g(x) = -x^2 - 7x - 10$ Max: $-2x - 7 = 0$ **OR/OF** $x = \frac{-(-7)}{2(-1)}$ $x = -\frac{7}{2}$ $D\left(-\frac{7}{2}\right) = -\left(-\frac{7}{2}\right)^2 - 7\left(-\frac{7}{2}\right) - 10 = 2{,}25$ $\therefore k = 2{,}25$ $\therefore h(x) = x + 7{,}25$	✓distance ✓ $-2x - 7 = 0$ ✓ $x = -\frac{7}{2}$ ✓ $k = 2{,}25$ ✓ $h(x) = x + 7{,}25$ (5)
		[14]

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QUESTION 4

4.1	$10 = a\left(\frac{1}{3}\right)^{-2} + 7$ $3 = 9a$ $\therefore a = \frac{1}{3}$	✓ subs (–2 ; 10) ✓ simplification ✓ answer (3)
4.2	$y = g(0)$ $y = \frac{1}{3} \times \left(\frac{1}{3}\right)^{0} + 7$ $y = \frac{22}{3} = 7{,}33$ $\therefore \left(0 ; \frac{22}{3}\right)$ ANSWER ONLY: FULL MARKS	✓ substitution of $x = 0$ ✓ answer (2)
4.3.1	Translation by 1 unit to the right and 7 units downwards	✓ 1 unit right ✓ 7 units downwards (2)
4.3.2	$h(x) = \left(\frac{1}{3}\right)^{x}$ ANSWER ONLY: FULL MARKS $h^{-1}: \quad x = \left(\frac{1}{3}\right)^{y}$ $y = \log_{\frac{1}{3}}(x)$ **OR/*OF*** $y = -\log_{3}(x)$	✓ swap x and y ✓ answer (2)
		[9]

QUESTION 5

5.1	$g(x)=\frac{a}{x+2}+q$ Subs (1 ; 0): $0=\frac{a}{1+2}+q$ $0=a+3q$ Subs $\left(0\ ;-\frac{1}{2}\right)$ $-\frac{1}{2}=\frac{a}{0+2}+q$ $-1=a+2q$ Solving simultaneously: $q=1$ $a=-3$ $\therefore g(x)=\frac{-3}{x+2}+1$	✓ $g(x)=\frac{a}{x+2}+q$ ✓ $0=a+3q$ ✓ $-1=a+2q$ ✓ solving simultaneously ✓ $q=1$ ✓ $a=-3$ (6)
5.2	$y \in \mathrm{R};\ y \neq 1$ **OR/*OF*** $(-\infty\,;1)$ or $(1\,;\infty)$ **OR/*OF*** $y<1$ or $y>1$	✓ answer (1)
5.3	$y-1=1(x+2)$ **OR/*OF*** $1=1(-2)+c$ $y=x+3$ ANSWER ONLY: FULL MARKS $c=3$ $y=x+3$	✓ $m=1$ ✓ subs point $(-2\ ;1)$ ✓ answer (3)
5.4	K′ (–3 ; 4)	✓ x-value ✓ y-value (2)
		[12]

QUESTION 6

6.1	$f(x) = -x^2 - 6x + 7$ $f'(x) = -2x - 6$ $-2x - 6 = 0$ **OR/*OF*** $x = -\frac{(-6)}{2(-1)}$ $x = -3$ E(–3 ; 16) ANSWER ONLY: FULL MARKS	✓ method ✓ x-value ✓ y-value (3)
6.2	$k = f(-5)$ $k = -(-5)^2 - 6(-5) + 7$ $\therefore k = 12$	✓ answer (A) (1)
6.3	C(0 ; 7) D(–5 ; 12) $m_{CD} = \frac{12 - 7}{-5 - 0}$ $m_{CD} = -1$ Equation of CD: $y = -x + 7$	✓ coordinates of C ✓ substitution ✓ m ✓ answer (4)
6.4	$-2x - 6 = -1$ $-2x = 5$ $x = -\frac{5}{2}$ $y = f\left(\frac{-5}{2}\right) = -\left(\frac{-5}{2}\right)^2 - 6\left(\frac{-5}{2}\right) + 7 = \frac{63}{4} = 15,75$ $\therefore \text{P}\left(-\frac{5}{2}; \frac{63}{4}\right)$	✓ $f'(x) = -2x - 6$ ✓ equating to -1 ✓ x-value ✓ y-value (A) (4)
6.5	Point by symmetry: (–1 ; 12) $-5 < x < -1$ **OR/*OF*** $-x^2 - 6x + 7 > 12$ $-x^2 - 6x - 5 > 0$ $x^2 + 6x + 5 < 0$ $(x + 1)(x + 5) < 0$ $-5 < x < -1$ ANSWER ONLY: FULL MARKS	✓ –1 ✓ answer (2) ✓ –1 ✓ answer (2)
		[14]

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QUESTION 7/*VRAAG* 7

7.1	$A = P(1-i)^n$ $8\,337{,}75 = 13\,000(1-i)^6$ $i = 7{,}14\%$	✓ substitution in correct formula ✓✓ answer (3)
7.2	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $80\,000 = \frac{x\left[\left(1+\frac{8{,}6}{1200}\right)^{36} - 1\right]}{\frac{8{,}6}{1200}}$ $x = \text{R}1\,955{,}78$ Thandi's total = 1955,78 × 36 = R 70 408,08 Eric's total = 1402,31 × 48 = R 67 310,88 Difference = 70 409,08 – 67 310,88 = R3 097,20	✓ i ✓substitution into correct formula ✓answer ✓ answer (4)
7.3	$A = P(1+i)^n$ $A = 225000\left(1+\frac{0{,}09}{12}\right)^3$ $A = R230100{,}5637...$ $225000\left(1+\frac{0{,}09}{12}\right)^3 = \frac{5\,500\left[1-\left(1+\frac{0{,}09}{12}\right)^{-n}\right]}{\frac{0{,}09}{12}}$ $0{,}3137734959... = 1-\left(1+\frac{0{,}09}{12}\right)^{-n}$ $\left(1+\frac{0{,}09}{12}\right)^{-n} = 0{,}6862265041...$ $-n = \log_{\left(1+\frac{0{,}09}{12}\right)} 0{,}6862265041...$ $n = 50{,}394375...$ $n = 51$	✓ substitution in correct formula ✓ answer ✓ substitution ✓ simplification ✓ use of logs ✓ answer (6)
		[13]

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QUESTION 6/*VRAAG 6*

6.1.1	$A = P(1+i)^n$ $A = 150\,000(1+0{,}065)^5$ A = R205 513	✓ substitution into the correct formula ✓ answer (2)
6.1.2	$A = P(1-in)$ $A = 150\,000(1-0{,}09\times 5)$ $A = 150\,000 - 67\,000$ A = R82 500	✓ substitution into the correct formula ✓ answer (2)
6.1.3	SF = A – T = 205 513 – 82 500 = R123 013 $F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $x = \frac{F \times i}{(1+i)^n - 1}$ $x = \frac{123\,013 \times \frac{0{,}0785}{12}}{\left[\left(1+\frac{0{,}0785}{12}\right)^{59} - 1\right]\left(1+\frac{0{,}0785}{12}\right)}$ = R1 704,01	✓ answer ✓ $i = \frac{0{,}0785}{12}$ ✓ 59 and $\left(1+\frac{0{,}0785}{12}\right)$(A) ✓ answer (A) (4)
6.2	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $200\,000 = \frac{6\,000\left[1-\left(1+\frac{0{,}0525}{4}\right)^{-4n}\right]}{\frac{0{,}0525}{4}}$ $\frac{7}{16} = 1-\left(1+\frac{0{,}0525}{4}\right)^{-4n}$ $\frac{9}{16} = \left(\frac{1621}{1600}\right)^{-4n}$ $-4n = \frac{\log\frac{9}{16}}{\log\left(\frac{1621}{1600}\right)}$ $-4n = -44{,}1243\ldots$ $n = 11{,}03$ years	✓ substitution into correct formula ✓ simplification ✓ use of logs ✓ $-4n = -44{,}1243\ldots$ ✓ $n = 11{,}03$ years (5)
		[13]

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QUESTION 7

7.1	$A = P(1+i)^n$ $2 = 1\left(1+\frac{0,085}{4}\right)^{4n}$ $4n = \log_{\left(1+\frac{0,085}{4}\right)} 2$ $n = 8,24$ years	✓ 2 ✓ $\frac{0,085}{4}$ } In correct formula ✓ use of logs ✓ answer in years (4)
7.2.1	$A = P(1-i)^n$ $180\ 000 = 500\ 000(1-i)^5$ $\frac{9}{25} = (1-i)^5$ $\sqrt[5]{\frac{9}{25}} = 1-i$ $i = 0,1848068...$ $r = 18,48\%$	✓ subs into correct formula ✓ simplification ✓ $i = 0,1848\ldots$ ✓ answer (4)
7.2.2	$A = P(1+i)^n$ $A = 500\ 000(1+0,063)^5$ A = R678 635,11	✓ subs into correct formula ✓ answer (2)
7.2.3	Sinking Fund = 678 635,11 – 180 000 = R 498 635,11 $498\ 635,11 = \frac{x\left[\left(1+\frac{0,1025}{12}\right)^{58}-1\right]\left(1+\frac{0,1025}{12}\right)^3}{\frac{0,1025}{12}}$ x = R6 510,36	✓ value of sinking fund ✓ $\frac{0,1025}{12}$ ✓ $n = 58$ (A) ✓ $\left(1+\frac{0,1025}{12}\right)^3$ ✓ answer (A) (5)
		[15]

May/June 2021

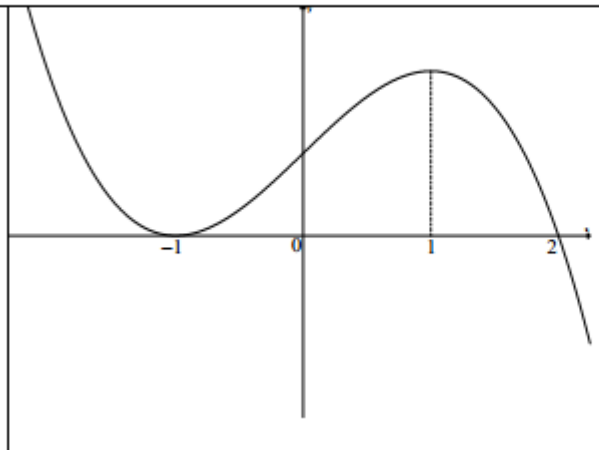
QUESTION/*VRAAG* 8

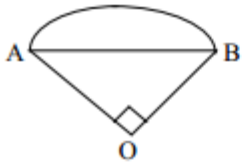
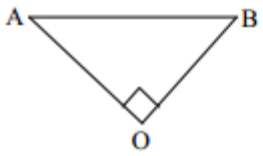
8.1	$f(x)=3x^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{3(x+h)^2-3x^2}{h}$ $f'(x)=\lim_{h\to 0}\frac{3x^2+6xh+3h^2-3x^2}{h}$ $=\lim_{h\to 0}\frac{6xh+3h^2}{h}$ $=\lim_{h\to 0}\frac{h(6x+3h)}{h}$ $=6x$	✓ substitution ✓ expansion ✓ simplification ✓ $\lim_{h\to 0}\frac{h(6x+3h)}{h}$ ✓ $6x$ (5)
8.2.1	$f(x)=x^2-3+9x^{-2}$ $f'(x)=2x-18x^{-3}$	✓ $9x^{-2}$ ✓ $2x$ ✓ $-18x^{-3}$ (3)
8.2.2	$g(x)=(\sqrt{x}+3)(\sqrt{x}-1)$ $g(x)=x+2x^{\frac{1}{2}}-3$ $g'(x)=1+x^{-\frac{1}{2}}$	✓ x ✓ $2x^{\frac{1}{2}}$ ✓ 1 ✓ $x^{-\frac{1}{2}}$ (4)
		[12]

QUESTION/*VRAAG* 9

9.1	$f'(x)=6x^2+6x-12$ $6x^2+6x-12=0$ $x^2+x-2=0$ $(x+2)(x-1)=0$ $x=-2$ or $x=1$ $y=20$ or $y=-7$ $\therefore$ A(–2 ; 20) and B(1 ; –7)	✓ $6x^2+6x-12$ ✓ $=0$ ✓factors ✓ x -values ✓ y -values (5)
9.2	$f''(x)=12x+6$ $12x+6>0$ $12x>-6$ $x>-\frac{1}{2}$ **OR/OF** $x=\frac{-2+1}{2}=-\frac{1}{2}$ $\therefore x>-\frac{1}{2}$	✓$12x+6$ ✓ $f''(x)>0$ ✓ $x>-\frac{1}{2}$ (3) **OR/OF** ✓ $x=-\frac{1}{2}$ ✓✓ $x>-\frac{1}{2}$ (3)
9.3	$f'(2)=24$ Equation of the tangent: $y-4=24(x-2)$ $y=24x-44$	✓ $f'(2)$ ✓ 24 ✓ equation (3)
		[11]

QUESTION/*VRAAG* 10

10.1		✓ $x=-1$ and $x=2$ ✓ TP at $x=-1$ ✓ TP at $x=1$ ✓ shape (4)

10.2.1	 Area of segment $= \frac{1}{4}$ Area of big circle $= \frac{1}{4}\pi\ (x-x^2)^2$  Area triangle ABO counted $=$ Area $\Delta = \frac{1}{2}(x-x^2)^2$ Area of shaded region $= \frac{1}{4}\pi(x-x^2)^2 - \frac{1}{2}(x-x^2)^2$ $= \frac{\pi-2}{4}(x-x^2)^2$ $= \left(\frac{\pi-2}{4}\right)(x^2-2x^3+x^4)$	✓✓ $\frac{1}{4}\pi(x-x^2)^2$ ✓ Area $\Delta = \frac{1}{2}(x-x^2)^2$ ✓ subtract areas ✓ common factor (5)

10.2.2	Area of shaded region $= \frac{(\pi-2)}{4}(x^4-2x^3+x^2)$ $\frac{dA}{dx} = \left(\frac{\pi-2}{4}\right)(4x^3-6x^2+2x)$ $4x^3-6x^2+2x=0$ $x(2x^2-3x+1)=0$ $x(2x-1)(x-1)=0$ $x \neq 0$ or $x=\frac{1}{2}$ or $x \neq 1$	✓ $\left(\frac{\pi-2}{4}\right)(4x^3-6x^2+2x)$ ✓ factors ✓ $x=0; x=1; x=\frac{1}{2}$ ✓ $x=\frac{1}{2}$ (4)
		[13]

May/June 2019

QUESTION/*VRAAG* 7

7.1	$f(x)=x^2+2$ $f(x+h)=(x+h)^2+2$ $=x^2+2xh+h^2+2$ $f(x+h)-f(x)=x^2+2xh+h^2+2-(x^2+2)$ $=2xh+h^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2}{h}$ $=\lim_{h\to 0}\frac{h(2x+h)}{h}$ $=\lim_{h\to 0}(2x+h)$ $=2x$ **OR/*OF*** $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{x^2+2xh+h^2+2-(x^2+2)}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2}{h}$ $=\lim_{h\to 0}\frac{h(2x+h)}{h}$ $=\lim_{h\to 0}(2x+h)$ $=2x$	✓ $x^2+2xh+h^2+2$ ✓ $\lim_{h\to 0}\frac{2xh+h^2}{h}$ ✓ $\lim_{h\to 0}\frac{h(2x+h)}{h}$ ✓ answer (4) **OR/*OF*** ✓ $x^2+2xh+h^2+2$ ✓ $\lim_{h\to 0}\frac{2xh+h^2}{h}$ ✓ $\lim_{h\to 0}\frac{h(2x+h)}{h}$ ✓ answer (4)
7.2.1	$y=4x^3+2x^{-1}$ $\frac{dy}{dx}=12x^2-2x^{-2}$	✓ $+2x^{-1}$ ✓ $12x^2$ ✓ $-2x^{-2}$ (3)
7.2.2	$y=4\sqrt[3]{x}+(3x^3)^2$ $=4x^{\frac{1}{3}}+9x^6$ $\frac{dy}{dx}=\frac{4}{3}x^{-\frac{2}{3}}+54x^5$	✓ $4x^{\frac{1}{3}}$ ✓ $9x^6$ ✓ $\frac{4}{3}x^{-\frac{2}{3}}$ ✓ $54x^5$ (4)
7.3	Point of contact: (1 ; 5) $m=2$ $y-y_1=m(x-x_1)$ or $y=2x+c$ $y-5=2(x-1)$ $5=2+c$ $c=3$ $y=2x+3$ $y=2x+3$	✓ $m=2$ ✓ substitution of (1 ; 5) ✓ answer (3) **[14]**

QUESTION/*VRAAG* 8

8.1	$h(x)=-2(x+\frac{3}{2})(x-1)(x+3)$ $h(x)=-(2x+3)(x^2+2x-3)$ $h(x)=-2x^3-7x^2+9$ **OR/*OF*** $h(x)=-(2x+3)(x-1)(x+3)$ $h(x)=-(2x+3)(x^2+2x-3)$ $h(x)=-2x^3-7x^2+9$	✓✓$-2(x+\frac{3}{2})(x-1)(x+3)$ ✓correct simplification (3) **OR/*OF*** ✓✓ $-(2x+3)(x-1)(x+3)$ ✓ correct simplification (3)
8.2	$h'(x)=-6x^2-14x$ $-6x^2-14x=0$ $-2x(3x+7)=0$ $x=0$ or $x=-\frac{7}{3}$	✓ first derivative ✓ $=0$ ✓ both answers (3)
8.3	$x<-\frac{7}{3}$ or $x>0$ **OR/*OF*** $x\in\left(-\infty;-\frac{7}{3}\right)\cup(0;\infty)$	✓✓ answer (2) **OR/*OF*** ✓✓ answer (2)

8.4	$y=4x+7$ $-6x^2-14x=4$ $0=6x^2+14x+4$ $0=3x^2+7x+2$ $0=(3x+1)(x+2)$ $x=-\frac{1}{3}$ or $x=-2$	✓ $y=4x+7$ ✓ $h'(x)=4$ ✓ standard form ✓ both answers (4) **[12]**

QUESTION/*VRAAG* 9

9.1	Volume of Sphere $=\frac{4}{3}\pi(8)^3$ or $=\frac{2048\pi}{3}$ or $=2144{,}66$	✓ answer (1)
9.2	$r^2+x^2=8^2$ (Pythagoras) $r^2=64-x^2$	✓ substitution or reason Pythagoras (1)
9.3	$V_{cone}=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi\left(64-x^2\right)(8+x)$ $=\frac{\pi}{3}\left(512+64x-8x^2-x^3\right)$ $\frac{dV}{dx}=\frac{64\pi}{3}-\frac{16\pi}{3}x-\frac{3\pi}{3}x^2$ $0=64-16x-3x^2$ $0=(8-3x)(x+8)$ $x=\frac{8}{3}\qquad x\neq -8$ $\frac{V_{cone}}{V_{sphere}}=\frac{\frac{1}{3}\pi\left(\frac{512}{9}\right)\left(\frac{32}{3}\right)}{\frac{2048\pi}{3}}$ $=\frac{8}{27}=0{,}3$	✓ $h=8+x$ ✓ $\frac{1}{3}\pi\left(64-x^2\right)(8+x)$ ✓ expansion ✓ $\frac{dV}{dx}=\frac{64\pi}{3}-\frac{16\pi}{3}x-\frac{3\pi}{3}x^2$ ✓ $x=\frac{8}{3}$ ✓ volume of the cone ✓ $\frac{8}{27}$ or 0,3 (7) **[9]**

May/June 2024

QUESTION8/*VRAAG* 8

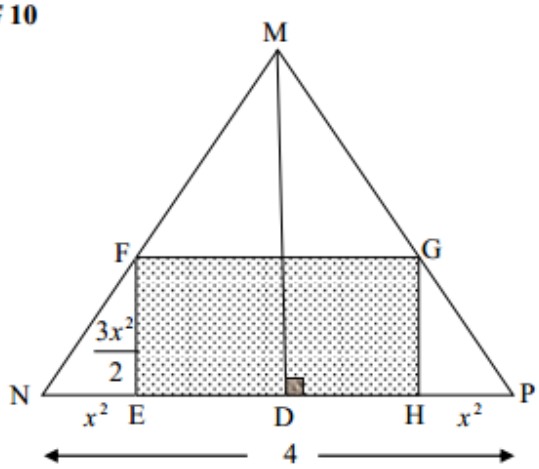
8.1	$f(x)=\frac{1}{x}$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{\frac{1}{(x+h)}-\frac{1}{x}}{h}$ $f'(x)=\lim_{h\to 0}\frac{x-(x+h)}{x(x+h)}\times\frac{1}{h}$ $f'(x)=\lim_{h\to 0}\frac{-h}{x(x+h)}\times\frac{1}{h}$ $f'(x)=\lim_{h\to 0}\frac{-1}{x(x+h)}$ $f'(x)=-\frac{1}{x^2}$ **OR/*OF*** $f(x)=\frac{1}{x}$ $f(x+h)=\frac{1}{x+h}$ $f(x+h)-f(x)=-\frac{h}{x(x+h)}$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-h}{x(x+h)}\times\frac{1}{h}$ $f'(x)=\lim_{h\to 0}\frac{-1}{x(x+h)}$ $f'(x)=-\frac{1}{x^2}$	✓ $f(x+h)=\frac{1}{x+h}$ ✓ $\frac{x-(x+h)}{x(x+h)}\times\frac{1}{h}$ ✓ $\frac{-h}{x(x+h)}\times\frac{1}{h}$ ✓ $\frac{-1}{x(x+h)}$ ✓answer (5) **OR/*OF*** ✓ $f(x+h)=\frac{1}{x+h}$ ✓ $f(x+h)-f(x)=-\frac{h}{x(x+h)}$ ✓ $\frac{-h}{x(x+h)}\times\frac{1}{h}$ ✓ $\frac{-1}{x(x+h)}$ ✓answer (5)

8.2.1	$\frac{d}{dx}\left(\sqrt{4x^6}+\sqrt{2}.x^2\right)$ $=\frac{d}{dx}\left(2x^3+\sqrt{2}.x^2\right)$ $=6x^2+2\sqrt{2}x$	✓ $2x^3$ ✓ $6x^2$ ✓ $2\sqrt{2}x$ (3)
8.2.2	$g(x)=\frac{3x^4-4x^2+6}{x^2}$ $g(x)=3x^2-4+6x^{-2}$ $g'(x)=6x-12x^{-3}$	✓ $3x^2-4+6x^{-2}$ ✓ $6x$ ✓ $-12x^{-3}$ (3)
8.3	$f(x)=3x^2+bx+c$ $f'(x)=6x+b$ $f'(1)=6+b=9$ $\therefore b=3$ $f(1)=3+3+c=0$ $c=-6$ $\therefore f(x)=3x^2+3x-6$	✓ $f'(1)=6+b=9$ ✓ $b=3$ ✓ $f(1)=3+3+c=0$ ✓ $c=-6$ (4)
		[15]

QUESTION9/*VRAAG* 9

9.1	$f(x)=ax^3+bx^2+cx-5$ $-5=a(0+1)^2(0-5)$ $-5=-5a$ $a=1$ $f(x)=(x+1)(x+1)(x-5)$ $f(x)=(x^2+2x+1)(x-5)$ $f(x)=x^3-3x^2-9x-5$ $\therefore\ b=-3$ and $c=-9$	✓ substitution of x-intercepts ✓ simplification ✓ simplification (3)
9.2	$f(x)=x^3-3x^2-9x-5$ $f'(x)=3x^2-6x-9$ $x^2-2x-3=0$ $(x-3)(x+1)=0$ $x=3$ or $x=-1$ Minimum value at $x=3$	✓ $f'(x)=3x^2-6x-9$ ✓ $f'(x)=0$ ✓factors ✓ $x=3$ (4)
9.3	$f''(x).f(x)>0$ Point of inflection: $x=1$ $x<1\ ;\ x\neq -1$ or $x>5$	✓ $x=1$ ✓ $x<1\ ;\ x\neq -1$ ✓ $x>5$ (3)
9.4	$-32<-t<-5$ $5<t<32$ **OR/*OF*** Shift up more than 5 units and less than 32 units $\therefore 5<t<32$	✓-32 ✓$-32<-t<-5$ ✓ $5<t<32$ (3) **OR/*OF*** ✓more than 5 units ✓less than 32 units ✓ $5<t<32$ (3)
		[13]

QUESTION 10/*VRAAG* 10



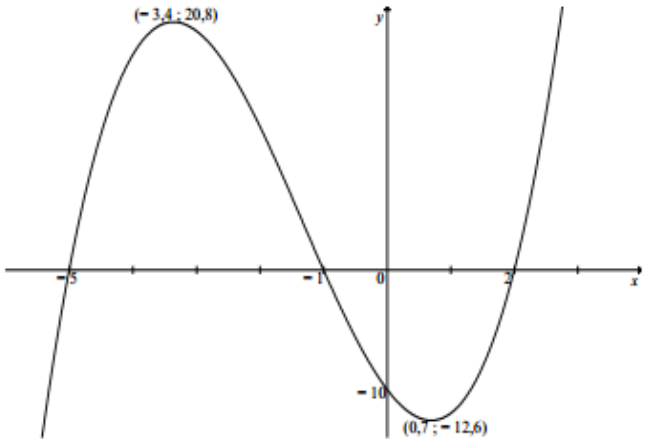
10.1	$\frac{NE}{EF}=\frac{2}{3}=\frac{x^2}{b}$ $3x^2=2b$ $\therefore b=\frac{3x^2}{2}$ $EH=4-2x^2$ Area EFGH $=(4-2x^2)\left(\frac{3x^2}{2}\right)$ $A(x)=6x^2-3x^4$ **OR/OF** In ΔDMP: $\tan P=\frac{3}{2}$ In ΔHGP: $\tan P=\frac{GH}{x^2}$ $\frac{GH}{x^2}=\frac{3}{2}$ $\therefore b=\frac{3x^2}{2}$ $EH=4-2x^2$ Area EFGH $=(4-2x^2)\left(\frac{3x^2}{2}\right)$ $A(x)=6x^2-3x^4$	✓ $\frac{NE}{EF}=\frac{2}{3}=\frac{x^2}{b}$ ✓ $\therefore b=\frac{3x^2}{2}$ ✓ $EH=4-2x^2$ ✓ $(4-2x^2)\left(\frac{3x^2}{2}\right)$ (4) **OR/OF** ✓ $\frac{GH}{x^2}=\frac{2}{3}$ ✓ $\therefore b=\frac{3x^2}{2}$ ✓ $EH=4-2x^2$ ✓ $(4-2x^2)\left(\frac{3x^2}{2}\right)$ (4)
10.2	$A(x)=6x^2-3x^4$ $A'(x)=12x-12x^3=0$ $12x(1-x^2)=0$ $\therefore x\neq 0$ or $x=-1$ or $x=1$ $\therefore$ max area: $A(1)=6(1)^2-3(1)^4=3\text{ cm}^2$	✓ $12x-12x^3=0$ ✓ values of x ✓ correct substitution ✓ answer (4)
		[8]

May/June 2023

QUESTION 7/*VRAAG 7*

7.1	$f(x)=-2x^2-1$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-2(x+h)^2-1-\left(-2x^2-1\right)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-2x^2-4xh-2h^2-1+2x^2+1}{h}$ $=\lim_{h\to 0}\frac{-4xh-2h^2}{h}$ $=\lim_{h\to 0}\frac{h(-4x-2h)}{h}$ $=-4x$ **OR/*OF*** $f(x+h)=-2(x+h)^2-1$ $f(x+h)=-2x^2-4xh-2h^2-1$ $f(x+h)-f(x)=-2x^2-4xh-2h^2-1+2x^2+1$ $f(x+h)-f(x)=-4xh-2h^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{-4xh-2h^2}{h}$ $=\lim_{h\to 0}\frac{h(-4x-2h)}{h}$ $=-4x$	✓substitution into the correct formula ✓$-2x^2-4xh-2h^2-1$ ✓$-4xh-2h^2$ ✓common factor ✓answer (5) **OR/*OF*** ✓$-2x^2-4xh-2h^2-1$ ✓$-4xh-2h^2$ ✓substitution into the correct formula ✓common factor ✓answer (5)
7.2.1	$f(x)=-2x^3+3x^2$ $f'(x)=-6x^2+6x$	✓$-6x^2$ ✓$+6x$ (2)
7.2.2	$y=2x+\frac{1}{\sqrt{4x}}$ $y=2x+\frac{1}{2}x^{-\frac{1}{2}}$ $\frac{dy}{dx}=2-\frac{1}{4}x^{-\frac{3}{2}}$	✓$\frac{1}{2}$ ✓$x^{-\frac{1}{2}}$ ✓2 ✓$-\frac{1}{4}x^{-\frac{3}{2}}$ (4)
7.3	$x<1$	✓✓ answer (2)
		[13]

QUESTION 8/*VRAAG* 8

8.1	$y=-10$	✓ answer (1)
8.2	$f(x)=x^3+4x^2-7x-10$ $f(2)=2^3+4(2)^2-7(2)-10=0$	✓substitution of $x=2$ ✓ $f(2)=0$ (2)
8.3	$f(x)=(x-2)(x^2+6x+5)$ $f(x)=(x-2)(x+5)(x+1)$	✓ $(x-2)$ ✓ $(x+5)$ ✓ $(x+1)$ (3)
8.4		✓ x- intercepts ✓ y-intercept ✓ sketching the graph with turning points in 2nd and 4th quadrant (3)
8.5.1	$x\in(-3,4\,;0,7)$ **OR/*OF*** $-3,4<x<0,7$	✓✓ $x\in(-3,4\,;0,7)$ (2)
8.5.2	$f(x)=x^3+4x^2-7x-10$ $f'(x)=3x^2+8x-7$ $f''(x)=6x+8=0$ $\therefore x=-\frac{8}{6}=-\frac{4}{3}=-1,33$ **OR/*OF*** $\frac{-3,4+0,7}{2}=-1,35=-1,35$	✓ $f''(x)=6x+8$ ✓ answer (2) **OR/*OF*** ✓ substitution ✓ answer (2)
8.5.3	$x\le-3,4$ or $-1,33\le x\le 0,7$ **OR/*OF*** $x\in(-\infty\,;-3,4]\cup[-1,33\,;0,7]$	✓ $x\le-3,4$ (A) ✓✓ $-1,33\le x\le 0,7$ (A 0,7) (3)
		[16]

QUESTION 9/VRAAG 9

9.1	Perimeter of the square $= 12-6x$ Side length of square $= \frac{12-6x}{4} = \frac{6-3x}{2} = 3-\frac{3}{2}x$	✓ $12-6x$ ✓ answer (2)
9.2	$V = \left(\frac{6-3x}{2}\right)^2 (4x)$ $= \left(\frac{36-36x+9x^2}{4}\right)(4x)$ $= 36x - 36x^2 + 9x^3$ $V(x) = 36x - 36x^2 + 9x^3$ $V'(x) = 36 - 72x + 27x^2$ $36 - 72x + 27x^2 = 0$ $9x^2 - 24x + 12 = 0$ $3x^2 - 8x + 4 = 0$ $(3x-2)(x-2) = 0$ $x = \frac{2}{3}$ or $x = 2$ $V\left(\frac{2}{3}\right) = 36\left(\frac{2}{3}\right) - 36\left(\frac{2}{3}\right)^2 + 9\left(\frac{2}{3}\right)^3$ $= \frac{32}{3}\text{m}^3 = 10{,}67\text{m}^3$	✓ $\left(\frac{6-3x}{2}\right)^2 (4x)$ ✓ $\left(\frac{36-36x+9x^2}{4}\right)$ ✓ $36x - 36x^2 + 9x^3$ ✓ V' ✓ $V' = 0$ ✓ values ✓ answer (7)
		[9]

May/June 2022

QUESTION/*VRAAG* 8

8.1	$f(x)=-x^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-(x+h)^2+x^2}{h}$ $f'(x)=\lim_{h\to 0}\frac{-x^2-2xh-h^2+x^2}{h}$ $=\lim_{h\to 0}\frac{-2xh-h^2}{h}$ $=\lim_{h\to 0}\frac{h(-2x-h)}{h}$ $=\lim_{h\to 0}(-2x-h)$ $\therefore f'(x)=-2x$ **OR/*OF*** $f(x)=-x^2$ $f(x+h)=-(x+h)^2=-x^2-2xh-h^2$ $f(x+h)-f(x)=-x^2-2xh-h^2-(-x^2)=-2xh-h^2$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{-2xh-h^2}{h}$ $=\lim_{h\to 0}\frac{h(-2x-h)}{h}$ $=\lim_{h\to 0}(-2x-h)$ $\therefore f'(x)=-2x$	✓ substitution into formula ✓ $-\left(x^2+2xh+h^2\right)$ ✓ $-2xh-h^2$ ✓ $-2x-h$ ✓answer (5) **OR/*OF*** ✓ $-x^2-2xh-h^2$ ✓ $-2xh-h^2$ ✓substitution into the formula ✓ $-2x-h$ ✓answer (5)
8.2.1	$f(x)=4x^3-5x^2$ $f'(x)=12x^2-10x$	✓ $12x^2$ (A) ✓ $-10x$ (A) (2)
8.2.2	$D_x\left[\frac{-6\sqrt[3]{x}+2}{x^4}\right]$ $=D_x\left[\frac{-6(x)^{\frac{1}{3}}}{x^4}+\frac{2}{x^4}\right]$ $=D_x\left[-6x^{-\frac{11}{3}}+2x^{-4}\right]$ $=22x^{-\frac{14}{3}}-8x^{-5}$	✓ $x^{\frac{1}{3}}$ ✓ $-6x^{-\frac{11}{3}}+2x^{-4}$ ✓ $22x^{-\frac{14}{3}}$ ✓ $-8x^{-5}$ (4)
		[11]

QUESTION/*VRAAG* 9

9.1	$f(x)=(x+t)^2(x-3)$ $-3=(0+t)^2(0-3)$ $1=t^2$ $t=\pm 1$ $\therefore t=1$ $f(x)=(x+1)^2(x-3)$ $f(x)=(x^2+2x+1)(x-3)$ $f(x)=x^3-x^2-5x-3$	✓ $f(x)=(x+t)^2(x-3)$ ✓ subs (0 ; –3) ✓ t ✓ $f(x)=(x+1)^2(x-3)$ ✓ expansion (5)
9.2	$f'(x)=3x^2-2x-5$ $0=3x^2-2x-5$ $0=(x+1)(3x-5)$ $x=-1$ or $x=\frac{5}{3}$ $\text{N}\left(\frac{5}{3};-\frac{256}{27}\right)=(1,67;-9,48)$	✓ $f'(x)=3x^2-2x-5$ ✓ $=0$ ✓ factors ✓ x-value ($x>0$) ✓ y-value (A) (5)
9.3.1	$x<3$; $x\neq -1$ **OR/*OF*** $x<-1$ or $-1<x<3$ **OR/*OF*** $(-\infty;-1)$ or $(-1;3)$	✓ $x<3$ ✓ $x\neq -1$ (2) **OR/*OF*** ✓ $x<-1$ ✓ $-1<x<3$ (2) **OR/*OF*** ✓ $(-\infty;-1)$ ✓ $(-1;3)$ (2)
9.3.2	$x<-1$ or $x>\frac{5}{3}$ OR/OF $x\leq -1$ or $x\geq\frac{5}{3}$ **OR/*OF*** $(-\infty;-1)$ or $\left(\frac{5}{3};\infty\right)$ OR/OF $(-\infty;-1]$ or $\left[\frac{5}{3};\infty\right)$	✓ $x<-1$ ✓ $x>\frac{5}{3}$ (2) **OR/*OF*** ✓ $(-\infty;-1)$ ✓ $\left(\frac{5}{3};\infty\right)$ (2)
9.3.3	$f''(x)>0$ $6x-2>0$ $x>\frac{1}{3}$ or $\left(\frac{1}{3};\infty\right)$ **OR/OF** $\frac{\frac{5}{3}+(-1)}{2}=\frac{1}{3}$ $x>\frac{1}{3}$ or $\left(\frac{1}{3};\infty\right)$ ANSWER ONLY: FULL MARKS	✓ $6x-2$ ✓ $\frac{1}{3}$ ✓ $x>\frac{1}{3}$ (3) **OR/OF** ✓ substitution ✓ $\frac{1}{3}$ ✓ $x>\frac{1}{3}$ (3)

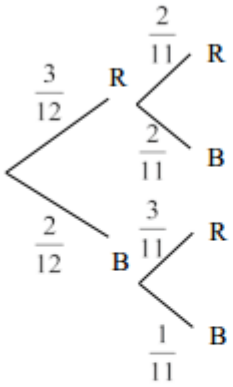
9.4	Distance $= x^3 - x^2 - 5x - 3 - (3x^2 - 2x - 5)$ $= x^3 - 4x^2 - 3x + 2$ $\frac{d\text{Distance}}{dx} = 3x^2 - 8x - 3$ $0 = 3x^2 - 8x - 3$ $0 = (3x + 1)(x - 3)$ $x = 3$ or $x = -\frac{1}{3}$ Max distance $= \left(-\frac{1}{3}\right)^3 - 4\left(-\frac{1}{3}\right)^2 - 3\left(-\frac{1}{3}\right) + 2$ $= \frac{68}{27} = 2{,}52$	✓ $x^3 - 4x^2 - 3x + 2$ ✓ $\frac{d\text{Distance}}{dx} = 3x^2 - 8x - 3$ ✓ factors ✓ x-values ✓ $x = -\frac{1}{3}$ ✓ answer (6)
		[23]

Counting Principle and Probability
May/June 2021
QUESTION/*VRAAG* 11

11.1	P(A) = 1 – P(not A) = 0,6 P(A and B) = P(A) × P(B) $= 0{,}6 \times 0{,}3$ $= \frac{9}{50}$ $= 0{,}18$	✓ 0,6 ✓ P(A and B) = P(A) × P(B) ✓ answer (A) (3)
11.2.1	$a = \frac{15}{150} = 0{,}1$	✓ $\frac{15}{150}$ (A) (1)
11.2.2	$m = 1 - 0{,}7 = 0{,}3$	✓ 0,3 (A) (1)
11.2.3	$0{,}24 + 0{,}14 + 0{,}02 + 0{,}12 + 0{,}1 + 2b = 0{,}7$ $2b = 0{,}08$ $b = 0{,}04$ $0{,}04 \times 150 = 6$	✓ addition ✓ simplification ✓ $b = 0{,}04$ ✓ 6 (4)
11.3.1	$9 \times 9 \times 8 = 648$	✓9 ✓9×8 (2)
11.3.2	$2 \times 8 \times 4 = 64$ $2 \times 8 \times 5 = 80$ Total number = 64 + 80 = 144	✓✓$2 \times 8 \times 4$ ✓$2 \times 8 \times 5$ ✓144 (A) (4)
		[15]

May/June 2019

QUESTION/*VRAAG* 10

10.1	 P(One Red and One Blue) = P(Red, Blue) + P(Blue, Red) $=\left(\frac{3}{12}\right)\times\left(\frac{2}{11}\right)+\left(\frac{2}{12}\right)\times\left(\frac{3}{11}\right)$ $=\frac{1}{11}$	✓ $\left(\frac{3}{12}\right)\times\left(\frac{2}{11}\right)$ ✓ $\left(\frac{2}{12}\right)\times\left(\frac{3}{11}\right)$ ✓ addition of products ✓ answer (4)
10.2.1	$a = 0{,}48 \times 250$ $a = 120$	✓ answer (1)
10.2.2	$b = 150$ P(S) × P(F) $=\frac{200}{250}\times\frac{150}{250}$ = 0,48 = P(S and F) These events are independent / *Hierdie gebeurtenisse is onafhanklik*	✓ b ✓ P(S) × P(F) ✓ $\frac{200}{250}$ and $\frac{150}{250}$ ✓ conclusion (with realistic probabilities) (4)
		[9]

QUESTION/*VRAAG* 11

11.1	10 × 9 = 90	✓✓ 10 × 9 (2)
11.2.1	10! =3 628 800	✓ 10! (1)
11.2.2	2! × 2! × 2! × 2! × 2! ×4! = 768	✓ 2! × 2! × 2! × 2! × 2! ✓ 4! ✓ 2! × 2! × 2! × 2! × 2!×4! or 768 (3) **[6]**

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QUESTION 11/*VRAAG* 11

11.1	$P(A) + P(B) = 0,52$ $0,4 + P(B) = 0,52$ $P(B) = 0,12$	✓ substitution ✓ answer (2)
11.2.1	$P(\text{sandwich}) = \frac{4}{25}$ **OR/*OF*** $0,02 + 0,01 + 0,04 + 0,09 = \frac{4}{25} = 0,16$	✓ answer (1) **OR/*OF*** ✓ answer (1)
11.2.2	$P(\text{at least two events}) = 0,02 + 0,01 + 0,03 + 0,04$ $= 0,1$	✓ $0,02 + 0,01 + 0,03 + 0,04$ ✓ answer (2)
11.2.3	$P(\text{not any}) = 1 - (0,1 + 0,04 + 0,09 + 0,2)$ $= 0,57$	✓ $1 - (0,1 + 0,04 + 0,09 + 0,2)$ ✓ answer (2)
11.3.1	$7! = 5040$	✓ $7!$ (1)
11.3.2	$P(\text{4 players alphabetically}) = \frac{1}{7 \times 6 \times 5 \times 4} = \frac{1}{840}$	✓ 1 ✓ 840 ✓ $\frac{1}{840}$ (3)

11.3.3	<table><tr><td></td><td>F</td><td></td><td>F</td><td></td><td>F</td><td></td><td>F</td><td></td></tr></table> F arrangements: 4! M arrangements: 5 options with 3 males = 5 × 4 × 3 4! × 5 × 4 × 3 = 1 440 **OR/*OF*** 10 Options: F **M** F **M** F **M** F **M** F **M** F **M** F F F F **M** F **M** F **M** F **M** F **M** F F **M** **M** F **M** F F **M** F **M** F **M** F F F **M** F **M** F F **M** F **M** **M** F F **M** F F **M** **M** F F **M** F **M** F **M** F F F **M** F **M** Hence 10 × 4! × 3! = 1440	✓4! ✓5 × 4 × 3 ✓1 440 (3) **OR/*OF*** ✓4! × 3! ✓× 10 ✓1 440 (3)
		[14]
		TOTAL/*TOTAAL*: 150

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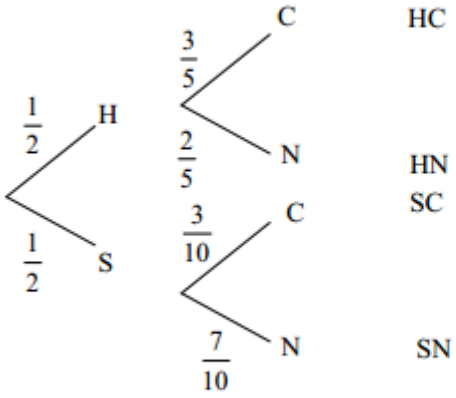
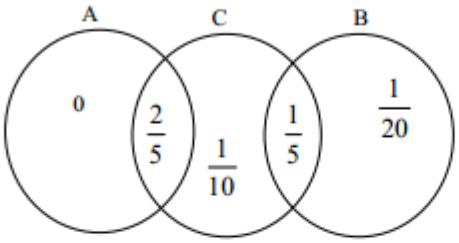
QUESTION 10/*VRAAG* 10

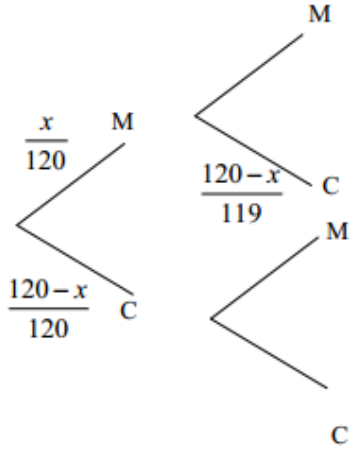
10.1.1	Event A Event B  	✓ Event A ✓ Event B Medication: for P(C) = $\frac{3}{5}$ ✓ Event B sugar pill: for P(NC) = $\frac{7}{10}$ (3)
10.1.2	P(Not Cured) = P(H) × P(NC) + P(S) × P(NC) $= \left(\frac{1}{2}\right)\left(\frac{2}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{7}{10}\right)$ $= \frac{11}{20} = 0{,}55$	✓substitution ✓answer (2)
10.2.1	P(A or B) = P(A) + P(B) – P(A and B) P(A and B) = $\frac{13}{20} - \frac{2}{5} - \frac{1}{4} = 0$ Events are mutually exclusive **OR/*OF*** P(A) + P(B) = $\frac{2}{5} + \frac{1}{4}$ $= \frac{13}{20}$ P(A or B) = P(A) + P(B) P(A and B) = 0 Events are mutually exclusive	✓ substitution ✓answer (P(A and B) = 0) (2) **OR/*OF*** ✓ substitution ✓answer (P(A and B) = 0) (2)

10.2.2	$P(B \text{ and } C) = \frac{1}{5} = 0,2$ $P(\text{ only } C) = \frac{7}{10} - \frac{2}{5} - \frac{1}{5} = \frac{1}{10} = 0,1$  	✓ $P(B \text{ and } C) = \frac{1}{5}$ (A) ✓ $\frac{7}{10} - \frac{2}{5} - \frac{1}{5}$ ✓ $\frac{1}{10}$ (3)
10.2.3	$P(\text{no event}) = 1 - \left(\frac{2}{5} + \frac{1}{10} + \frac{1}{5} + \frac{1}{20}\right) = \frac{1}{4} = 0,25$	✓1 – (P(A) or P(B) or P(C)) ✓answer (2)
10.3.1	$3! \times 5!$ $= 720$ Answer only: Full Marks	✓ $3!$ ✓ $3! \times 5!$ (A) (2)
10.3.2	$\frac{7! - 6! \times 2}{7!} = \frac{5}{7} = 0,71$ **OR/*OF*** $1 - \frac{6! \times 2}{7!}$ $= 1 - \frac{2}{7}$ $= \frac{5}{7} = 0,71$	✓ $7! - 6! \times 2$ ✓ denominator (7!) ✓ answer (3) **OR/*OF*** ✓ $6! \times 2$ ✓ denominator (7!) ✓ answer (3)
		[17]

May/June 2022

QUESTION/*VRAAG* 10

10.1.1	$7! = 5\,040$	✓✓ answer (2)
10.1.2	$4! \times 4!$ $= 576$ $P(\text{African flags together}) = \frac{576}{5040} \left(= \frac{4}{35} = 0,11\right)$	✓ $4!$ ✓ $4! \times 4!$ ✓ answer (A) (3)
10.2	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ $0,88 = 0,4 + P(B) - P(A \text{ and } B)$ $0,88 = 0,4 + P(B) - 0,4P(B)$ $0,48 = 0,6P(B)$ $P(B) = 0,8$	✓subs into rule ✓$P(A \text{ and } B) = 0,4P(B)$ ✓ answer (3)

10.3	First Passenger Second Passenger  Probability of first passenger choosing meat $= \frac{x}{120}$ Probability of second passenger choosing cheese $= \frac{120-x}{119}$ $\frac{x}{120} \times \frac{120-x}{119} = \frac{18}{85}$ $120x - x^2 = 3\,024$ $x^2 - 120x + 3\,024 = 0$ $(x-84)(x-36) = 0$ $x = 84 \quad \text{or} \quad x = 36$ $\therefore$ P(1st cheese) $= \frac{36}{120} = \frac{3}{10}$	✓ $\frac{x}{120}$ ✓ $\frac{120-x}{119}$ ✓ $\frac{x}{120} \times \frac{120-x}{119} = \frac{18}{85}$ ✓ $x = 84 \quad \text{or} \quad x = 36$ ✓ $\frac{3}{10}$ (5)
		[13]

Paper 2

Statistics and Regression

May/June 2021

QUESTION/VRAAG 1

1.1

26	13	3	18	12	34	24	58	16	10	15	69	20	17	40

1.1.1(a)	$\bar{x} = \frac{375}{15}$ $\bar{x} = 25\,\text{MB}$ Answer only: Full marks	✓ 375 ✓ answer (2)
1.1.1(b)	$\sigma = 17{,}65$ MB	✓answer (1)
1.1.2	$25 + 17{,}65 = 42{,}65$ $\therefore$ 2 days	✓ 42,65 ✓ 2 (2)
1.1.3	Overall $\bar{x} = \frac{80}{100} \times 25$ $= 20$ MB $\frac{375+x}{30} = 20$ $x = 600 - 375$ $= 225$ maximum total amount of data that Sam must use for the remainder of the month: **225** MB	✓ Overall $\bar{x} = 20$ ✓ $\frac{375+x}{30} = 20$ ✓ answer (3)

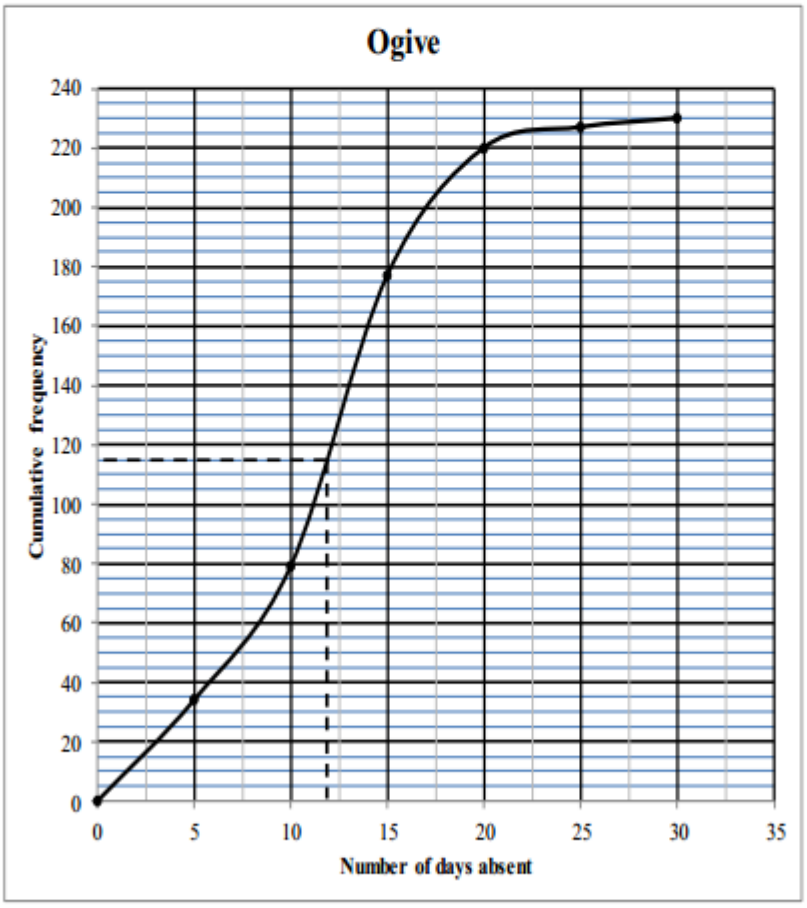
1.2

Wind speed in km/h (x)	2	6	15	20	25	17	11	24	13	22
Temperature in °C (y)	28	26	22	22	16	20	24	19	26	19

1.2.1	$a = 29{,}35$ $b = -0{,}46$ $\hat{y} = 29{,}35 - 0{,}46x$	✓ a ✓ b ✓ equation (3)
1.2.2	$y = 25{,}20$ °C (calculator) OR $\hat{y} = 29{,}35 - 0{,}46(9)$ $y = 25{,}21$ °C	✓✓ answer (2) ✓ substitution ✓ answer (2)
1.2.3	$b < 0$, indicating that as the wind speed increases the temperature decreases.	✓ interpretation (1)
		[14]

QUESTION/VRAAG 2

Number of days absent	Number of learners	Cumulative frequency
$0 \leq x < 5$	34	34
$5 \leq x < 10$	45	79
$10 \leq x < 15$	98	177
$15 \leq x < 20$	43	220
$20 \leq x < 25$	7	227
$25 \leq x < 30$	3	230

2.1	Modal class: $10 \leq x < 15$	✓ answer (1)
2.2	177 learners	✓ answer (1)
2.3	230 learners	✓ answer (1)
2.4		✓ grounding at (0; 0) ✓ shape ✓ upper limits ✓ All other points correct (4)
2.5	The median is at position 115. □value of median is 12 days (accept 11 – 14) Answer only: Full marks	✓ reading off at 115 ✓ answer (2)
		[9]

May/June 2019

QUESTION/*VRAAG* 1

1.1 45 children

✓ answer (1)

1.2 $\bar{x} = \frac{\sum fx}{n} = \frac{(4\times2)+(8\times10)+(12\times9)+(16\times7)+(20\times8)+(24\times7)+(28\times2)}{45}$

$\bar{x} = \frac{692}{45}$ OR $\bar{x} = 15{,}38$ minutes

Answer only: full marks

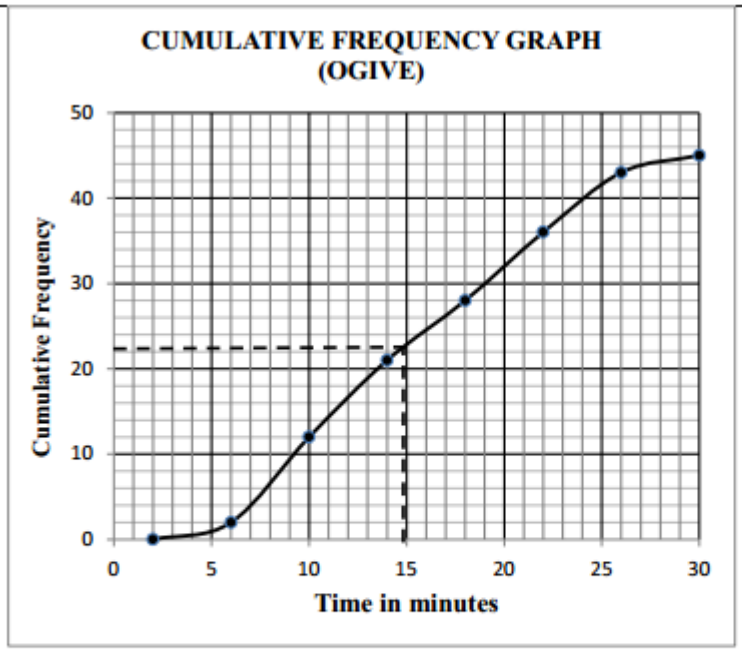
✓ 692
✓ answer (2)

1.3

Time taken (t) (in minutes)	Number of children	Cumulative frequency
$2 < t \leq 6$	2	2
$6 < t \leq 10$	10	12
$10 < t \leq 14$	9	21
$14 < t \leq 18$	7	28
$18 < t \leq 22$	8	36
$22 < t \leq 26$	7	43
$26 < t \leq 30$	2	45

✓ first 4 cum freq correct
✓ last 3 cum freq correct (2)

1.4



✓ plotting cum freq at upper limits correctly (all points)
✓ shape (smooth)
✓ grounding (2;0) (3)

1.5 On graph at the y-value of 22,5 or 23
Median = ± 15 minutes.

Answer only: full marks

✓ graph
✓ answer (2)

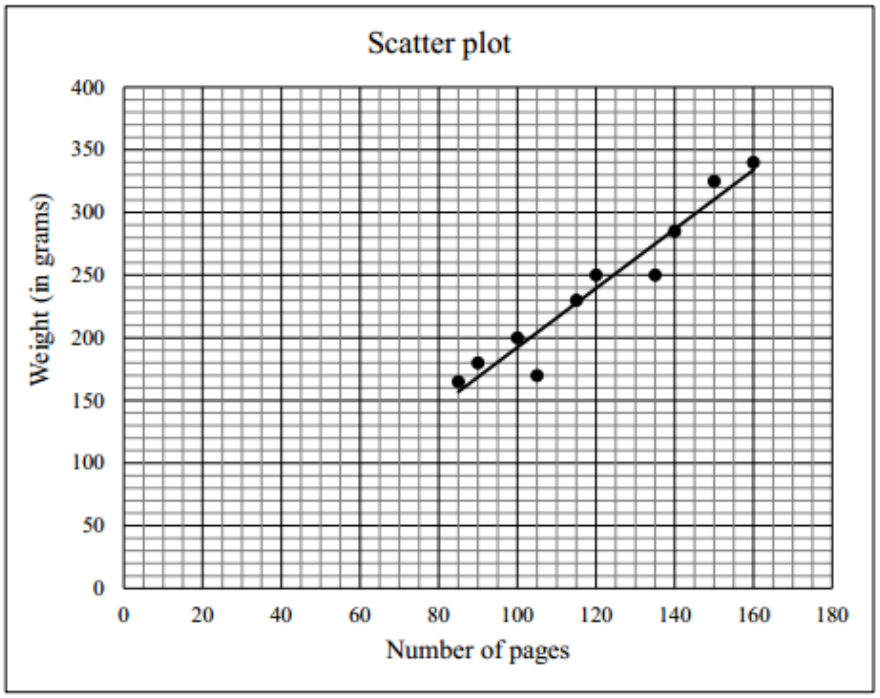
[10]

QUESTION/*VRAAG* 2

2.1	$a = 12{,}44$ $b = 0{,}98$ $y = 12{,}44 + 0{,}98x$ Answer only: full marks	✓ value of a ✓ value of b ✓ equation (3)
2.2.1	Percentage $= \frac{15}{50} \times 100$ $= 30\%$	✓ answer (1)
2.2.2	$\hat{y} = 12{,}44 + 0{,}98x$ $\hat{y} = 12{,}44 + 0{,}98(30)$ $\hat{y} = 41{,}84$ $= 42$ Answer only: full marks **OR** $\hat{y} = 41{,}87$ (if using calculator) $\hat{y} = 42$ **OR** $\hat{y} = \frac{21}{50}$	✓ substitution of 30 ✓ answer as integer (2) ✓ value of y ✓ answer as integer (2) ✓✓ answer (2)
2.3.1	standard deviation =13,88	✓✓ answer (2)
2.3.2	$x = 50{,}67 - 45{,}67$ $= 5\%$ Answer only: full marks	✓ $50{,}67 - 45{,}67$ ✓ answer (2)
		[10]

May/June 2024

QUESTION/*VRAAG* 1

1.1	$a = -43,72$ $b = 2,36$ $y = -43,72 + 2,36x$	✓ $a = -43,72$ ✓ $b = 2,36$ ✓ equation (3)
1.2		✓ any correct two points ✓ straight line joining the points for $x \in [85 ; 160]$ (2)
1.3	$y = -43,72 + 2,36(110)$ $y = 215,88$ **OR** $y = 215,90$ (calculator)	✓ substitution ✓ answer (2) ✓✓ answer (2)

1.4	$y = -43,72 + 2,36(130)$ $y = 263,08$ Percentage increase in weight $= \frac{263,08 - 215,88}{215,88} \times 100$ $= 21,86\%$ **OR** $y = 263,08$ Percentage $= \frac{263,08}{215,88} \times 100$ $= 121,86\ \%$ Percentage increase in weight $= 121,86 - 100 = 21,86$	✓ y-value ✓ difference between y-values ✓ +ve answer (3) ✓ y-value ✓ difference between % ✓ +ve answer (3)
		[10]

QUESTION/*VRAAG* 2

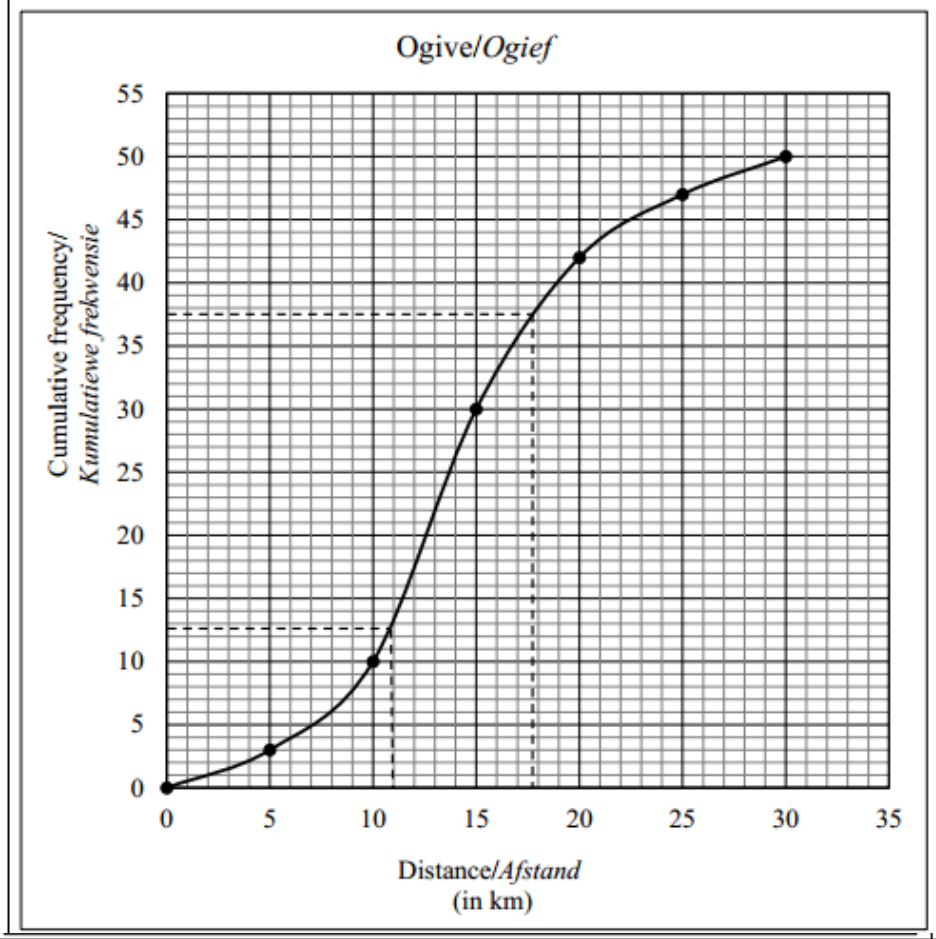
2.1

Distance (x km)	Frequency	Cumulative frequency
$0 \le x < 5$	3	3
$5 \le x < 10$	7	10
$10 \le x < 15$	20	30
$15 \le x < 20$	12	42
$20 \le x < 25$	5	47
$25 \le x < 30$	3	50

✓ 10
✓ all values correct

(2)

2.2



✓ grounding
✓ plotting a min of 3 points (cf at upper limits)
✓ smooth, increasing curve

(3)

2.3

$Q_3 = 17{,}8$
$Q_1 = 11$

$IQR = 6{,}8$

✓ Q_3 (accept between 17-19) and Q_1 (accept between 10-12,5)
✓ answer (accept 5-9)

(2)

2.4	$5 \le x < 10$	✓ $5 \le x < 10$ (1)
2.5	Estimated mean $= \frac{2,5(3)+7,5(11)+12,5(20)+17,5(8)+22,5(5)+27,5(3)}{50}$ $= \frac{675}{50}$ $= 13,5$ km	✓ new frequencies ✓ $\sum fx$ ✓ answer (3)
		[11]

May/June 2023

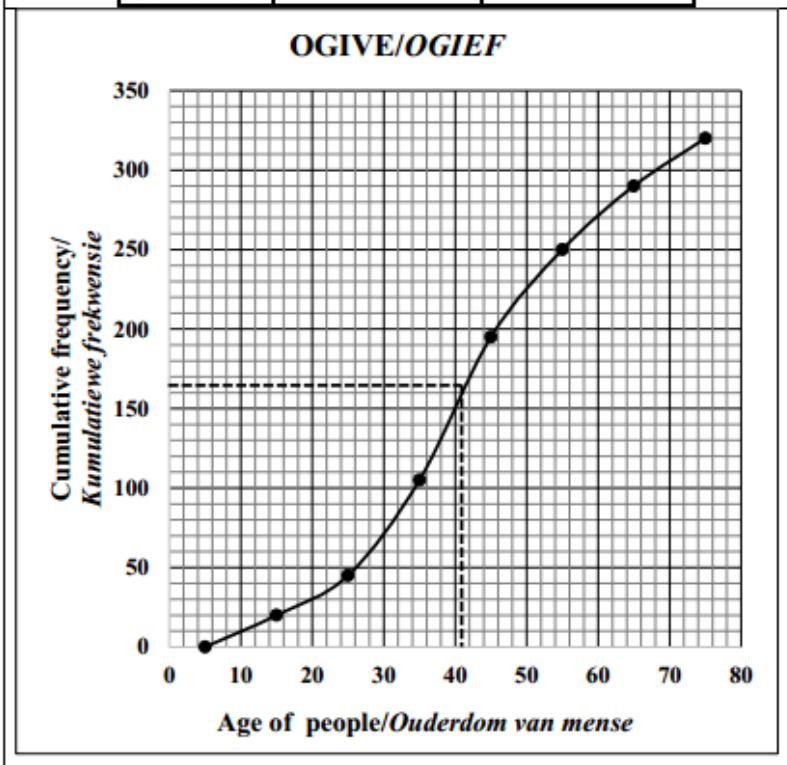
QUESTION/*VRAAG 1*

1.1.1	$a = 1730,22$ $b = 13,96$ $\hat{y} = 1730,22 + 13,96x$	✓ $a = 1730,22$ ✓ $b = 13,96$ ✓ equation (3)
1.1.2	$\hat{y} = 1730,22 + 13,96x$ $\hat{y} = 1730,22 + 13,96(28\,500)$ $\hat{y} = R399\,590,22$ **OR/*OF*** $\hat{y} =$ R399 599,64 (calc)	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.1.3	$r = 0,98002 \ldots$ $r = 0,98$	✓ answer (1)
1.1.4	There is a very strong positive correlation between the amount spent on advertising and sales. / *Daar is 'n baie sterk positiewe korrelasie tussen die bedrag spandeer op advertensie en die verkope.*	✓ strong positive (1)
1.2.1	$\bar{x} = \frac{1\,552195}{9}$ $\bar{x} = 172\,466,11$	✓ $\bar{x} = \frac{1\,552195}{9}$ ✓ answer (2)
1.2.2	$\sigma = 56950,09$	✓ answer (1)
1.2.3	$\bar{x} + \sigma$ $= 172\,466,11 + 56950,09$ $= 229\,416,20$ 2 years/*jaar*	✓ $\bar{x} + \sigma$ ✓ answer (2)
		[12]

QUESTION/*VRAAG 2*

2.1	$35 < x \leq 45$	✓ answer (1)
2.2	320 people/*mense*	✓ answer (1)
2.3		

AGE	NUMBER OF PEOPLE	CUMULATIVE FREQUENCY
$5 < x \leq 15$	20	20
$15 < x \leq 25$	25	45
$25 < x \leq 35$	60	105
$35 < x \leq 45$	90	195
$45 < x \leq 55$	55	250
$55 < x \leq 65$	40	290
$65 < x \leq 75$	30	320

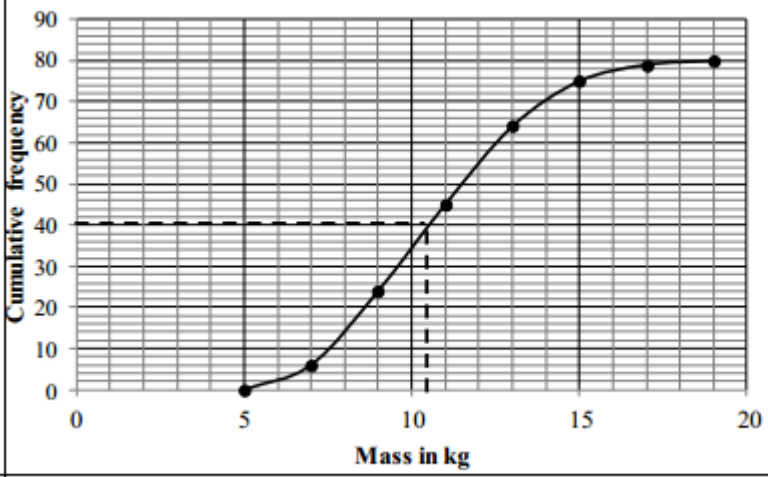


✓ cumulative frequency
✓ grounding
✓ plotting at upper limit
✓ shape
(4)

2.4	Median = 41	✓✓ answer (2)
		[8]

May/June 2022

QUESTION/*VRAAG* 1

1.1	Modal class: $9 < m \leq 11$	✓ answer (1)
1.2		✓ adding ✓ 80 (2)
1.3		✓ grounding (5 ; 0) ✓ points ✓ shape (3)
1.4	Median mass: 10,5 kg	✓✓ answer (2)

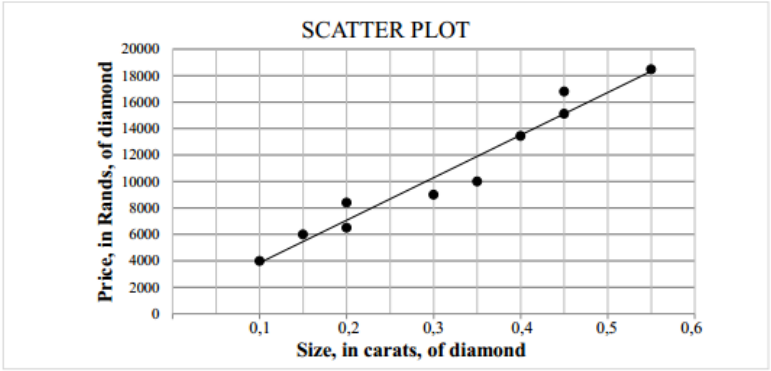
Mass (in kg)	Frequency	Cumulative frequency
$5 < m \leq 7$	6	6
$7 < m \leq 9$	18	24
$9 < m \leq 11$	21	45
$11 < m \leq 13$	19	64
$13 < m \leq 15$	11	75
$15 < m \leq 17$	4	79
$17 < m \leq 19$	1	80

1.5.1	$\bar{x} = \frac{(6\times6+18\times8+21\times10+19\times12+11\times14+4\times16+1\times18)}{80}$ $= \frac{854}{80}$ $= 10{,}68$ Answer only 2/2	✓ 854 ✓ answer (2)
1.5.2	Learners' bags are heavier than the stipulated international guideline. Estimated mean = 10,68 kg 10% of 80 kg = 8 kg 10,68 kg > 8 kg	✓ answer ✓ 8 kg (2)

	OR/ *OF*	
	Learners' bags are heavier than the stipulated international guideline.	✓ answer
	Estimated mean $= \frac{10,68}{80} \times 100$ $= 13,35\%$	✓ 13,35%
	$13,35\% > 10\%$	(2)
		[12]

QUESTION/*VRAAG* 2

Size, in carats, of diamond (x)	0,1	0,15	0,2	0,2	0,3	0,35	0,4	0,45	0,45	0,55
Price, in rands, of diamond (y)	4 000	6 000	6 500	8 400	9 000	10 000	13 440	15 120	16 800	18 480

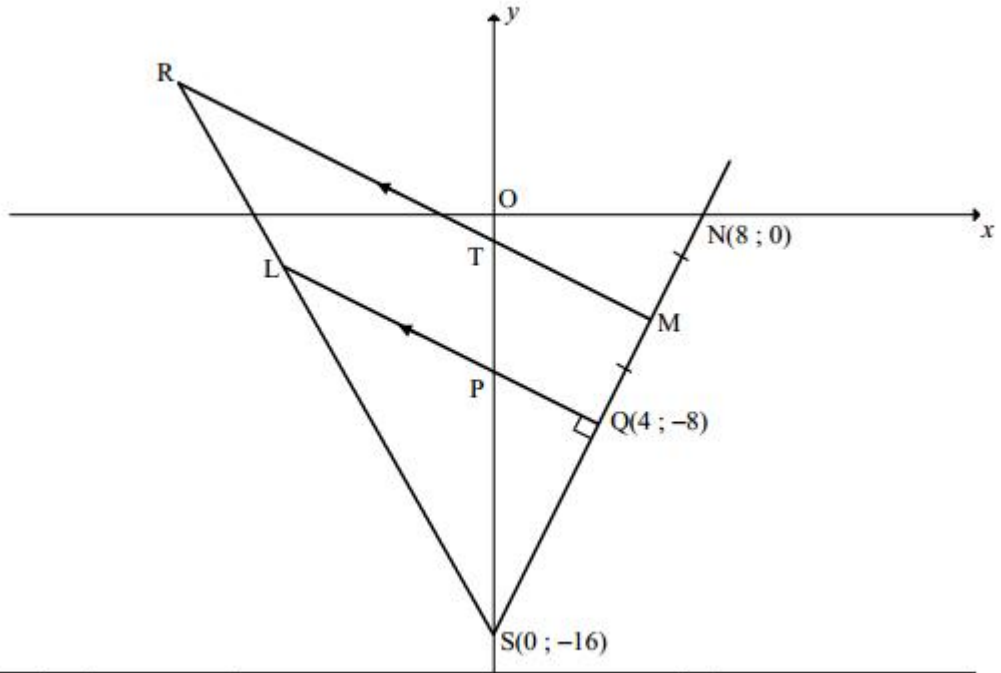


2.1	$a = 634,382\ldots$ $b = 32\,189,263\ldots$ $\hat{y} = 634,38 + 32189,26x$ Answer only 3/3	✓ a ✓ b ✓ equation (3)
2.2	$\hat{y} = 634,38 + 32189,26(0,25)$ $= R8\,681,70$ **OR/*OF*** $\hat{y} = R8\,681,70$ (if using calculator)	✓ substitution ✓ answer (2) ✓ ✓ answer (2)
2.3	Average price increase $= R\frac{32189,26}{20}$ per 0,05 carat $= R1\,609,46$ per 0,05 carat **OR/*OF*** Average price increase $= 0,05 \times 32189,26$ $= R1\,609,46$ per 0,05 carat **OR/*OF*** at 0,3: $\hat{y} = R10\,291,16$ $\therefore$ Average price increase $= 10\,291,16 - 8\,681,70$ $= R1\,609,46$ per 0,05 carat Answer only 2/2	✓ divide gradient by 20 ✓ answer (2) ✓ multiply gradient by 0,05 ✓ answer (2) ✓ Estimated price of a 0,3 carat diamond ✓ answer (2)
2.4	The point (0,35 ; 11500) is closer to the least squares regression line.	✓ reason (1)
		[8]

Analytical Geometry

May/June 2021

QUESTION/VRAAG 3

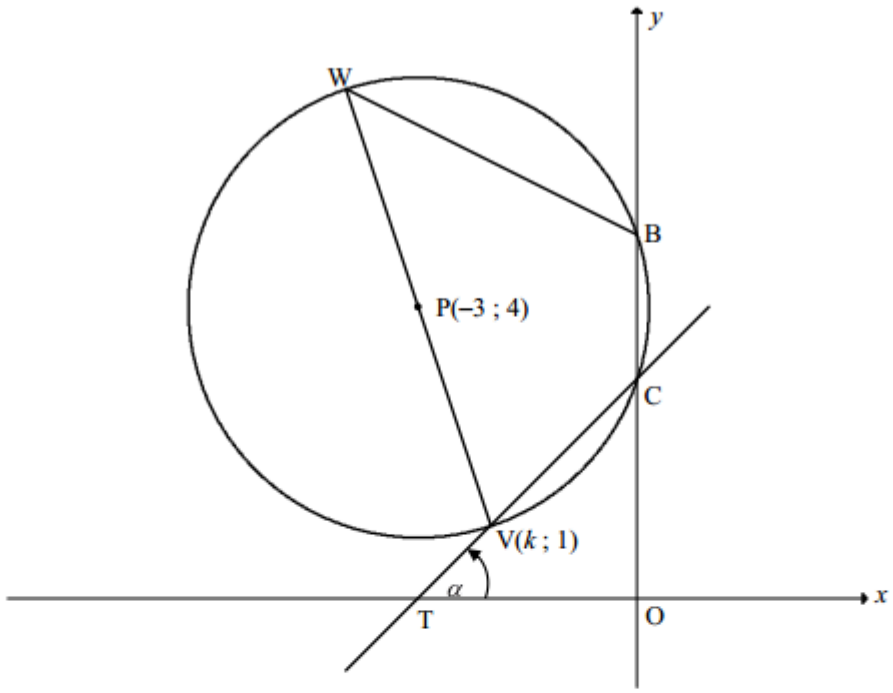


3.1	$M\left(\frac{4+8}{2};\frac{-8+0}{2}\right)$ $M(6;-4)$	✓ x_M ✓ y_M (2)
3.2	$m_{NS}=\frac{0-(-16)}{8-0}$ or $m_{NQ}=\frac{0-(-8)}{8-4}$ or $m_{QS}=\frac{-8-(-16)}{4-0}$ $=2$	✓ subst N and Q or N and Q or Q and S into gradient formula ✓ answer (2)
3.3	$m_{LQ}\times 2=-1$ [LQ ⊥ NS] $\therefore m_{LQ}=-\frac{1}{2}$ $-8=-\frac{1}{2}(4)+c$ **OR** $y+8=-\frac{1}{2}(x-4)$ $c=-6$ $y+8=-\frac{1}{2}x+2$ $\therefore y=-\frac{1}{2}x-6$	✓ m_{LQ} ✓ substitution of Q ✓ calculation of c or simplification (3)
3.4	OS is the radius of circle passing through S $(x-0)^2+(y-0)^2=(16)^2$ $x^2+y^2=256$ Answer only: Full marks	✓ identifying radius = 16 ✓ Equation of circle (2)

3.5	$m_{RM} = m_{LQ} = -\frac{1}{2}$ [RM ∥ LQ] $-4 = -\frac{1}{2}(6) + c$ **OR** $y + 4 = -\frac{1}{2}(x - 6)$ $c = -1$ $y + 4 = -\frac{1}{2}x + 3$ $\therefore y = -\frac{1}{2}x - 1$ T(0;−1)	✓ m_{RM} ✓ substitution of M(6;−4) ✓ coordinates of T (3)
3.6	T(0;−1), P(0;−6) and S(0;−16) ∴ PS = 10 units and TS = 15 units $\frac{LS}{RS} = \frac{PS}{TS} = \frac{2}{3}$ [prop theorem; RM ∥ LP] **OR** [line ∥ one side of Δ/*lyn* ∥ *een sy v Δ*] Answer only: Full marks **OR** M(6 ; −4), Q(4 ; −8) and S(0 ; −16) MS = $\sqrt{180} = 6\sqrt{5}$ and QS = $\sqrt{80} = 4\sqrt{5}$ $\frac{LS}{RS} = \frac{QS}{MS} = \frac{2}{3}$ [prop theorem; RM ∥ LQ] **OR** [line ∥ one side of Δ/*lyn* ∥ *een sy v Δ*] Answer only: Full marks	✓ PS = 10 units ✓ TS = 15 units ✓ answer (3) ✓ MS = $6\sqrt{5}$ units ✓ QS = $4\sqrt{5}$ units ✓ answer (3)
3.7	area of PTMQ = area of ΔTSM − area of ΔPSQ $= \frac{1}{2}.ST.\perp h_M - \frac{1}{2}.PS.\perp h_Q$ $= \frac{1}{2}(15)(6) - \frac{1}{2}(10)(4)$ = 45 − 20 = 25 square units **OR** TM = $\sqrt{45} = 3\sqrt{5} = 6{,}71$ MQ = $\sqrt{20} = 2\sqrt{5} = 4{,}47$ PQ = $\sqrt{20} = 2\sqrt{5} = 4{,}47$ area of trapezium PTMQ = $\frac{1}{2}(3\sqrt{5} + 2\sqrt{5})(2\sqrt{5})$ $= \frac{1}{2}(5\sqrt{5})(2\sqrt{5})$ = 25 square units	✓ area of ΔTSM − area of ΔPSQ ✓ area ΔTSM = 45 ✓ area ΔPSQ = 20 ✓ answer (4) ✓ TM = $3\sqrt{5}$ MQ = $2\sqrt{5}$ PQ = $2\sqrt{5}$ ✓ area of trapezium = $\frac{1}{2}$(sum of ∥sides)(height) ✓ substitute into formula ✓ answer (4)

	OR $MQ = \sqrt{20} = 2\sqrt{5}$ $PQ = \sqrt{20} = 2\sqrt{5}$ $TP = 5$ area of PTMQ= area of ΔMTP+ area of ΔPQM $\text{area of PTMQ} = \frac{1}{2}\text{TP} \times \perp h_M + \frac{1}{2}\text{MQ} \times \text{PQ}$ area of PTMQ= $\frac{1}{2}(5)\times 6 + \frac{1}{2}\left(2\sqrt{5}\right)\left(2\sqrt{5}\right)$ area of PTMQ= $10+15=25$	✓ area of ΔMTP + area of ΔPQM ✓ area ΔMTP = 10 ✓ area ΔPQM = 15 ✓ answer (4)
		[19]

QUESTION 4



4.1	$PV = r = \sqrt{10}$ $PV = \sqrt{(k-(-3))^2 + (1-4)^2} = \sqrt{10}$ $(PV)^2 = (k-(-3))^2 + (1-4)^2 = 10$ $k^2 + 6k + 9 + 9 = 10$ **OR** $(k+3)^2 + 9 = 10$ $k^2 + 6k + 8 = 0$ $(k+3)^2 = 1$ $(k+4)(k+2) = 0$ $k+3 = 1$ or $k+3 = -1$ $k = -4$ or $k = -2$ $\therefore k = -2$	✓ $PV = r = \sqrt{10}$ ✓ substitution into distance formula ✓ standard form ✓ factors ✓ answer (5)
4.2	$x^2 + 6x + y^2 - 8y + 15 = 0$ y-intercepts: $(0)^2 + 6(0) + y^2 - 8y + 15 = 0$ $(y-3)(y-5) = 0$ $y_C = 3$ or $y_B = 5$ $\therefore$ BC = 2 units	✓ $x = 0$ ✓ factors ✓ both values ✓ answer (4)

4.3.1	$m_{TC} = \frac{3-1}{0-(-2)}$ $= 1$ $\tan\alpha = 1$ $\therefore \alpha = 45°$ **OR** $y = mx + 3$ $1 = m(-2) + 3$ $m_{TC} = 1$ $\tan\alpha = 1$ $\therefore \alpha = 45°$	✓ substitution into gradient formula ✓ $\tan\alpha = 1$ ✓ answer (3) ✓ substitution into equation of a line ✓ $\tan\alpha = 1$ ✓ answer (3)
4.3.2	$B\hat{C}V = 135°$ [ext ∠ of Δ/*buite* ∠ v Δ] $\therefore V\hat{W}B = 45°$ [opp ∠s of cyclic quad/*teenoorst.* ∠*e v kvh*] Answer only: Full marks **OR** $T\hat{C}O = 45°$ [∠s of Δ/∠*e v* Δ] $\therefore V\hat{W}B = 45°$ [ext ∠s of cyclic quad/*buite* ∠ *v kvh*] Answer only: Full marks	✓ $B\hat{C}V = 135°$ ✓ answer (2) ✓ $T\hat{C}O = 45°$ ✓ answer (2)
4.4.1	Q(−3;−2)	✓ x_Q ✓ y_Q (2)
4.4.2	$(x+3)^2 + (y+2)^2 = 10$	✓ LHS ✓ RHS (2)
4.4.3	$x = -2$ or $x = -4$	✓ $x = -2$ ✓ $x = -4$ (2)
		[20]

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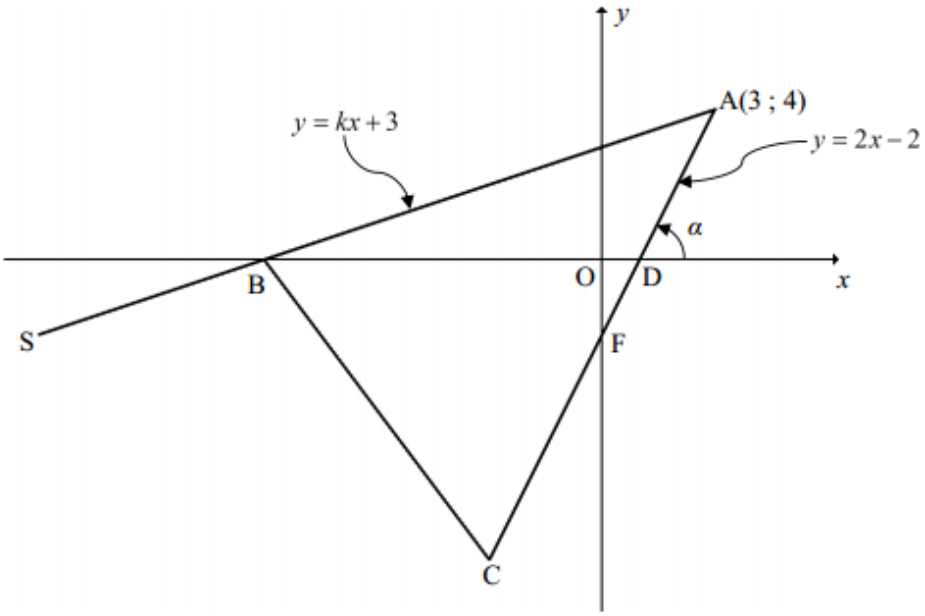
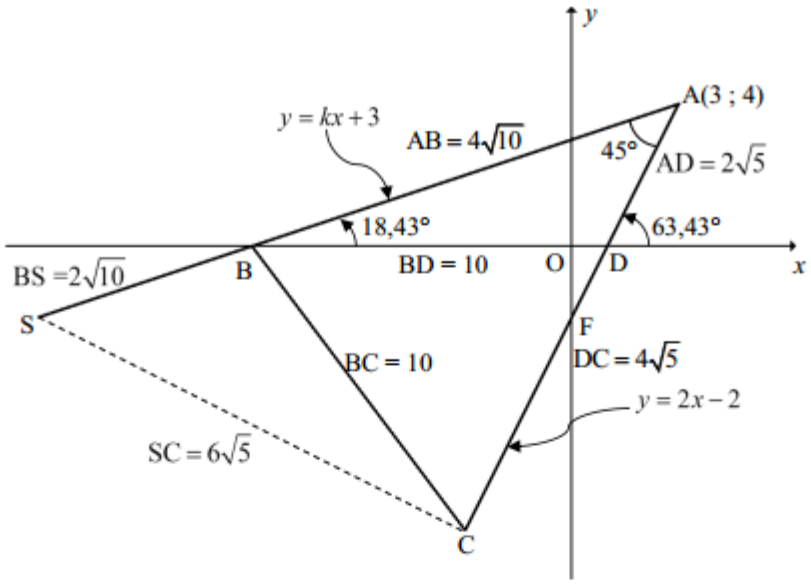
QUESTION/*VRAAG* 3



3.1	$y = kx + 3$ $4 = k(3) + 3$ $3k = 1$ $\therefore k = \frac{1}{3}$ **OR** y-intercept of AB: (0 ; 3) $m_{AB} = \frac{4-3}{3-0}$ $= \frac{1}{3}$ $\therefore k = \frac{1}{3}$	✓ substitution (3 ; 4) (1) ✓ substitution (0 ; 3) (1)
3.2	$0 = \frac{1}{3}x + 3$ $-3 = \frac{1}{3}x$ $x = -9$ $B(-9\ ;\ 0)$	✓ $y = 0$ ✓ answer (2)

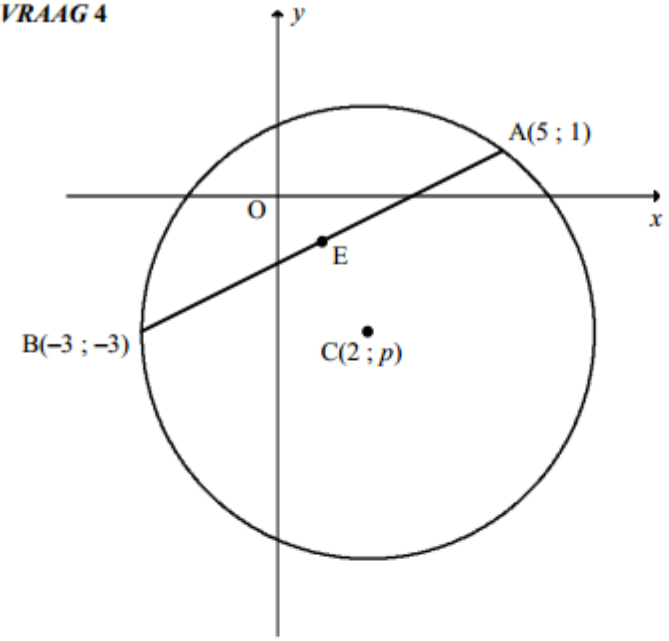
3.3	$F(0;-2)$ $F\left(\frac{x+3}{2};\frac{y+4}{2}\right)$ $\frac{x+3}{2}=0 \qquad \frac{y+4}{2}=-2$ $x=-3 \qquad y=-8$ $C(-3;-8)$ **OR** by translation $F(0;-2)$ $A\to F(x;y)\to(x-3;y-6)$ $F\to C(0;-2)\to(0-3;-2-6)=(-3;-8)$	✓ $F(0;-2)$ ✓ $\frac{x+3}{2}=0$; $\frac{y+4}{2}=-2$ ✓ x-value ✓ y-value (4) ✓ $F(0;-2)$ ✓ $(x-3;y-6)$ ✓ x-value ✓ y-value (4)
3.4	$m_{BC}=\frac{0-(-8)}{-9-(-3)}$ OR $m_{BC}=\frac{-8-0}{-3-(-9)}$ $m_{BC}=-\frac{4}{3}$ $y=-\frac{4}{3}x+c$ $(-2)=-\frac{4}{3}(-15)+c$ $c=-22$ $y=-\frac{4}{3}x-22$ **OR** $m_{BC}=\frac{0-(-8)}{-9-(-3)}$ OR $m_{BC}=\frac{-8-0}{-3-(-9)}$ $m_{BC}=-\frac{4}{3}$ $y-y_1=-\frac{4}{3}(x-x_1)$ $y-(-2)=-\frac{4}{3}(x-(-15))$ $y+2=-\frac{4}{3}x-20$ $y=-\frac{4}{3}x-22$	✓ substitution of B and C into the gradient formula ✓ m_{BC} ✓ $m_{line}=m_{BC}$ ✓ substitution of $S(-15;-2)$ ✓ equation (5) ✓ substitution into the gradient formula ✓ m_{BC} ✓ $m_{line}=m_{BC}$ ✓ substitution of $S(-15;-2)$ ✓ equation (5)



3.5	$\tan\alpha = m_{AC} = 2$ $\alpha = 63,43°$ $\tan A\hat{B}D = m_{AS} = \frac{1}{3}$ $A\hat{B}D = 18,43°$ $B\hat{A}C = \alpha - A\hat{B}D$ $B\hat{A}C = 63,43° - 18,43°$ $B\hat{A}C = 45°$ **OR** $AB = \sqrt{(-9-3)^2 + (0-4)^2}$ $AB = 4\sqrt{10}$ $BD = 10$ $AD = \sqrt{(3-1)^2 + (4-0)^2}$ $AD = 2\sqrt{5}$ $BD^2 = AB^2 + AD^2 - 2AB.AD\cos B\hat{A}C$ $(10)^2 = (4\sqrt{10})^2 + (2\sqrt{5})^2 - 2(4\sqrt{10})(2\sqrt{5})\cos B\hat{A}C$ $\cos B\hat{A}C = \frac{\sqrt{2}}{2}$ $B\hat{A}C = 45°$	✓ $\tan\alpha = m_{AC} = 2$ ✓ $\alpha = 63,43°$ ✓ $\tan A\hat{B}D = m_{AS} = \frac{1}{3}$ ✓ $A\hat{B}D = 18,43°$ ✓ answer (5) ✓ length of AB ✓ calculation of remaining 2 lengths ✓ substitution into cosine-rule ✓ rewriting in terms of $\cos B\hat{A}C$ ✓ answer (5)

3.6	A(3 ; 4) and S(−15 ; − 2)	
	$AS = \sqrt{(x_A - x_S)^2 + (y_A - y_S)^2}$	
	$AS = \sqrt{(3-(-15))^2 + (4-(-2))^2}$	✓ $AS = \sqrt{(3-(-15))^2 + (4-(-2))^2}$
	$AS = \sqrt{360} = 6\sqrt{10} = 18,97$	✓ length of AS
	$\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC} = \frac{\frac{1}{2}(BD)(\perp h)}{\frac{1}{2}(AS)(AC)\sin B\hat{A}C}$	
	$\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC} = \frac{\frac{1}{2}(10)(4)}{\frac{1}{2}\left(6\sqrt{10}\right)\left(6\sqrt{5}\right)\sin 45°}$	✓ Area ΔABD ✓ Area ΔASC
	$\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC} = \frac{2}{9}$	✓ answer (5)
	OR	
	$AS = \sqrt{(3-(-15))^2 + (4-(-2))^2}$	✓ $AS = \sqrt{(3-(-15))^2 + (4-(-2))^2}$
	$AS = \sqrt{360} = 6\sqrt{10} = 18,97$	✓ length of AS
	$AB = \sqrt{(-9-3)^2 + (0-4)^2} = 4\sqrt{10}$	
	$AD = \sqrt{(3-1)^2 + (4-0)^2} = 2\sqrt{5}$	
	$\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC} = \frac{\frac{1}{2}(AB)(AD)\sin \hat{A}}{\frac{1}{2}(AS)(AC)\sin \hat{A}}$	
	$= \frac{\frac{1}{2}\left(4\sqrt{10}\right)\left(2\sqrt{5}\right)\sin \hat{A}}{\frac{1}{2}\left(6\sqrt{10}\right)\left(6\sqrt{5}\right)\sin \hat{A}}$	✓ Area ΔABD ✓ Area ΔASC
	$= \frac{2}{9}$	✓ answer (5)
		[22]

QUESTION/*VRAAG* 4



4.1	$E\left(\frac{5+(-3)}{2};\frac{1+(-3)}{2}\right)$ $\therefore E(1;-1)$	✓ $x=1$ ✓ $y=-1$ (2)
4.2	$AB=\sqrt{(x_A-x_B)^2+(y_A-y_B)^2}$ $AB=\sqrt{(5-(-3))^2+(1-(-3))^2}$ $AB=\sqrt{80}=4\sqrt{5}=8,94$ units	✓ $AB=\sqrt{80}=4\sqrt{5}=8,94$ (1)
4.3	$m_{AB}=\frac{1-(-3)}{5-(-3)}$ $m_{AB}=\frac{1}{2}$ $\therefore m_{CE}=-2$ $[CE\perp AB]$ $E(1;-1)$ $y=-2x+c$ **OR** $y-y_1=-2(x-x_1)$ $(-1)=-2(1)+c$ $y-(-1)=-2(x-1)$ $c=1$ $y=-2x+1$ $y=-2x+1$	✓ $m_{AB}=\frac{1}{2}$ ✓ m_{CE} ✓ substitution of E ✓ equation (4)

4.4	$y=-2x+1$ $p=-2(2)+1$ $p=-3$ **OR** $m_{CE}=-2$ $\frac{p-(-1)}{2-1}=-2$ $p+1=-2$ $p=-3$	✓ substitution of C(2 ; p) into ⊥ bisector of AB (1) ✓ substitution of C and E into the gradient formula (1)
4.5	BC = r = 5 units $\therefore (x-2)^2+(y+3)^2=25$ $x^2-4x+4+y^2+6y+9=25$ $x^2+y^2-4x+6y-12=0$	✓ BC = r = 5 units ✓ $(x-2)^2+(y+3)^2$ ✓ r^2 ✓ $x^2-4x+4+y^2+6y+9=25$ (4)

4.6	$(x-2)^2+(y+3)^2=25$ **OR** $x^2+y^2-4x+6y-12=0$ $y=tx+8$ $(x-2)^2+(tx+8+3)^2=25$ **OR** $x^2+(tx+8)^2-4x+6(tx+8)-12=0$ $x^2-4x+4+t^2x^2+22tx+121-25=0$ **OR** $x^2+t^2x^2+16tx+64-4x+6tx+48-12=0$ $x^2(t^2+1)+x(22t-4)+100=0$ $\Delta<0$ $(22t-4)^2-4(t^2+1)(100)<0$ $484t^2-176t+16-400t^2-400<0$ $84t^2-176t-384<0$ $21t^2-44t-96<0$ $(7t-24)(3t+4)<0$ CV: $\frac{24}{7}$; $-\frac{4}{3}$  $\therefore t\in\left(-\frac{4}{3};\frac{24}{7}\right)$ **OR** $-\frac{4}{3}<t<\frac{24}{7}$	✓ substitution of $y=tx+8$ ✓ standard form ✓ $\Delta<0$ ✓standard form of Δ ✓ critical values ✓ answer (6)
		[18]

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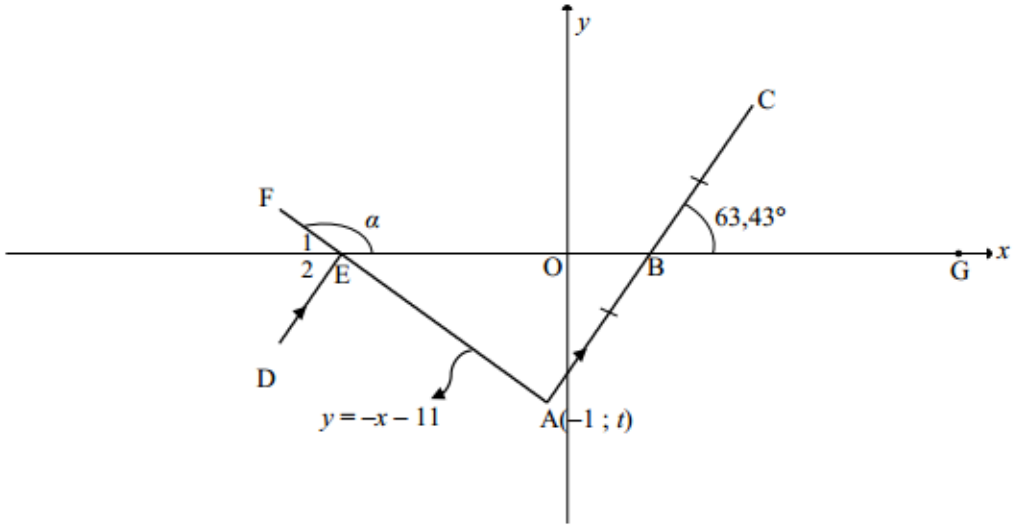
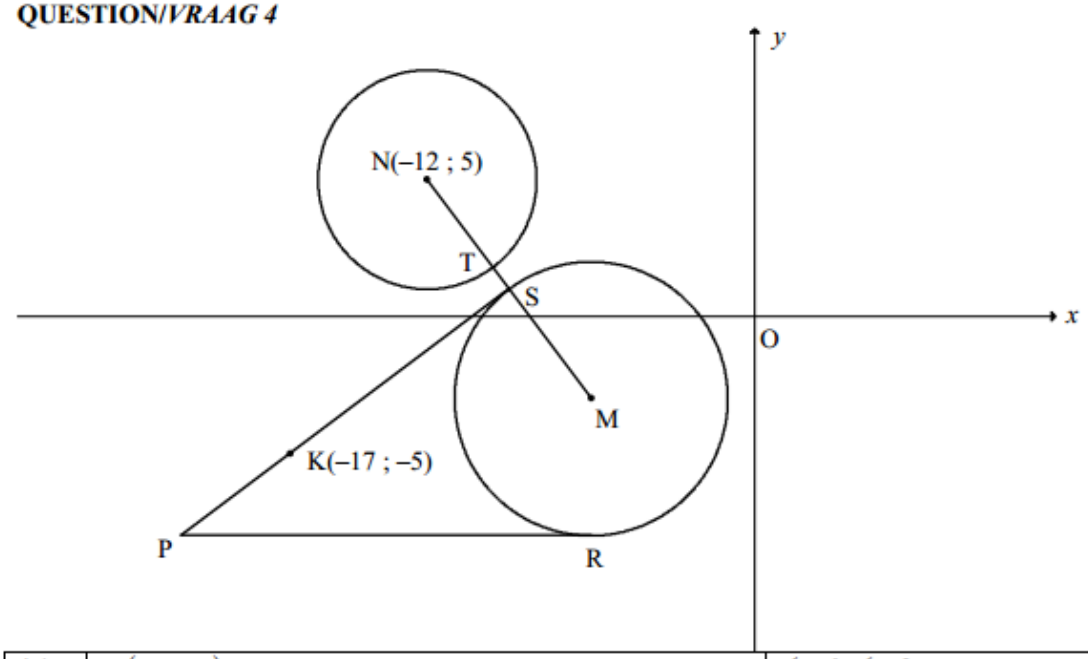
QUESTION/*VRAAG 3*



3.1.1	$y=-x-11$ A(–1 ; t) $t=-(-1)-11$ $t=-10$	✓ substitution ✓ value of t (2)
3.1.2	$\tan\alpha=-1$ $ref.\angle=45°$ $\therefore\alpha=135°$	✓ $\tan\alpha=-1$ ✓ 135° (2)
3.1.3	$\tan 63{,}43°=m_{AC}$ $m_{AC}=2$	✓ $\tan 63{,}43°=m_{AC}$ ✓ answer (2)
3.2	$m_{AC}=2$ A(–1 ; –10) $y=2x+k$ $-10=2(-1)+k$ $k=-8$ $y=2x-8$ **OR/*OF*** $y-y_1=2(x-x_1)$ $y-(-10)=2(x-(-1))$ $y=2x-8$	✓ substitution of m and A ✓ equation (2)

3.3.1	$y=2x-8$ $0=2x-8$ $x_B=4$ $\frac{x_C+(-1)}{2}=4$ $\qquad$ $\frac{y_C+(-10)}{2}=0$ $x_C=9$ $\qquad$ $y_C=10$ **OR/*OF*** by translation / *met translasie* $A\rightarrow B\ (x;y)\rightarrow(x+5;y+10)$ $B\rightarrow C\ (4;0)\rightarrow(4+5;0+10)=(9;10)$	✓ $x_B=4$ ✓ $x_C=9$ ✓ $y_C=10$ (3) ✓ $(x+5;y+10)$ ✓ $x_C=9$ ✓ $y_C=10$ (3)
3.3.2	$A\hat{B}E=63,43°$ [vert. opp ∠'s =] $\hat{E}_2=63,43°$ [corres. ∠'s, DE ‖ AB] $\hat{E}_1=45°$ [∠s on a str line] $F\hat{E}D=108,43°$ **OR/*OF*** $E\hat{A}B=135°-63,43°$ $E\hat{A}B=71,57°$ $D\hat{E}A=E\hat{A}B=71,57°$ $F\hat{E}D=108,43°$ **OR/*OF*** $A\hat{B}E=63,43°$ [vert. opp ∠'s] $D\hat{E}O=116,57°$ [co-int. ∠'s, DE ‖ AB] $F\hat{E}D=360°-(116,57°+135°)$ $=108,43°$	✓ $A\hat{B}E=63,43°$ ✓ $\hat{E}_1=45°$ ✓ $F\hat{E}D=108,43°$ (3) ✓ $E\hat{A}B=71,57°$ ✓ $D\hat{E}A=E\hat{A}B=71,57°$ ✓ $F\hat{E}D=108,43°$ (3) ✓ $A\hat{B}E=63,43°$ ✓ $D\hat{E}O=116,57°$ ✓ $F\hat{E}D=108,43°$ (3)
3.4	$y=0$ $x_E=-11$ $\frac{x_G+(-11)}{2}=4$ $x_G=19$ $(x-19)^2+y^2=15^2$ $(x-19)^2+y^2=225$	✓ $x_E=-11$ ✓ $x_G=19$ ✓ $(x-19)^2+y^2$ ✓ 225 (4)

QUESTION/*VRAAG 4*



4.1	$M(-6;-3)$	✓ –6 ✓ –3 (2)
4.2.1	$x^2+y^2+24x-10y+153=0$ $(x+12)^2+(y-5)^2=-153+144+25$ $(x+12)^2+(y-5)^2=16$ $r^2=16$ $r=4$ units	✓ $r^2=-153+144+25$ ✓ length of radius (2)
4.2.2	$NM=\sqrt{(-12-(-6))^2+(5-(-3))^2}$ NM = 10 units SM = 5 units $\therefore$ TS = 10 – 5 – 4 = 1 unit	✓ substitution into distance formula ✓ NM = 10 units ✓ SM = 5 units ✓ answer (4)
4.3.1	R(–6 ; –8) $y=-8$	✓ $y_R=-8$ ✓ answer (2)

4.3.2	$m_{NM} = \frac{5-(-3)}{-12-(-6)}$	✓ substitution
	$m_{NM} = -\frac{4}{3}$	✓ $m_{NM} = -\frac{4}{3}$
	$m_{tangent} = \frac{3}{4}$	✓ $m_{tangent} = \frac{3}{4}$
	$-5 = \frac{3}{4}(-17) + c$ **OR/*OF*** $y - y_1 = \frac{3}{4}(x - x_1)$	✓ substitution of m and N
	$c = \frac{31}{4}$ $y-(-5) = \frac{3}{4}(x-(-17))$	
	$y = \frac{3}{4}x + \frac{31}{4}$ $y = \frac{3}{4}x + \frac{31}{4}$	✓ equation (5)
	OR/OF	
	NS = SM = 5	✓ S midpoint
	$S\left(\frac{-12-6}{2}\ ;\ \frac{5-3}{2}\right)$	
	S (– 9 ; 1)	✓ coordinates of S
	$m_{SK} = \frac{1-(-5)}{-9+17}$	
	$= \frac{6}{8} = \frac{3}{4}$	✓ $m_{tangent} = \frac{3}{4}$
	$y+5 = \frac{3}{4}(x+17)$	✓ substitution of m and K(–17 ; –5) or S
	$y = \frac{3}{4}x + \frac{31}{4}$ *or* $y = \frac{3}{4}x + 7\frac{3}{4}$	✓ equation (5)
4.4.1	$-8 = \frac{3}{4}x + \frac{31}{4}$	✓ $-8 = \frac{3}{4}x + \frac{31}{4}$
	$-32 = 3x + 31$	
	$3x = -63$	
	$x = -21$	✓ $x = -21$
	P(–21 ; –8)	
	R(–6 ; –8)	
	PR = PS = 15 units [tangents from same point]	✓ PR = PS = 15 units
	MS = MR = 5 units	✓ MS = MR = 5 units
	Perimeter PSMR = 15 + 15 + 5 + 5	
	= 40 units	✓ answer (5)

4.4.2	$\frac{\text{area of } \Delta NPS}{\text{area of quadrilateral PSMR}}$ $\frac{\frac{1}{2}NS.SP}{\frac{1}{2}SP.MS+\frac{1}{2}MR.PR}$ $=\frac{\frac{1}{2}(5)(15)}{2\left(\frac{1}{2}\right)(5)(15)}$ $=\frac{1}{2}$ **OR** $\Delta NPS \equiv \Delta SPM \equiv \Delta MPR$ $\frac{\text{area of } \Delta NPS}{\text{area of quadrilateral PSMR}}$ $=\frac{1}{2}$	✓ substitution ✓ answer (2) ✓ congruent ✓ answer (2)
		[22]

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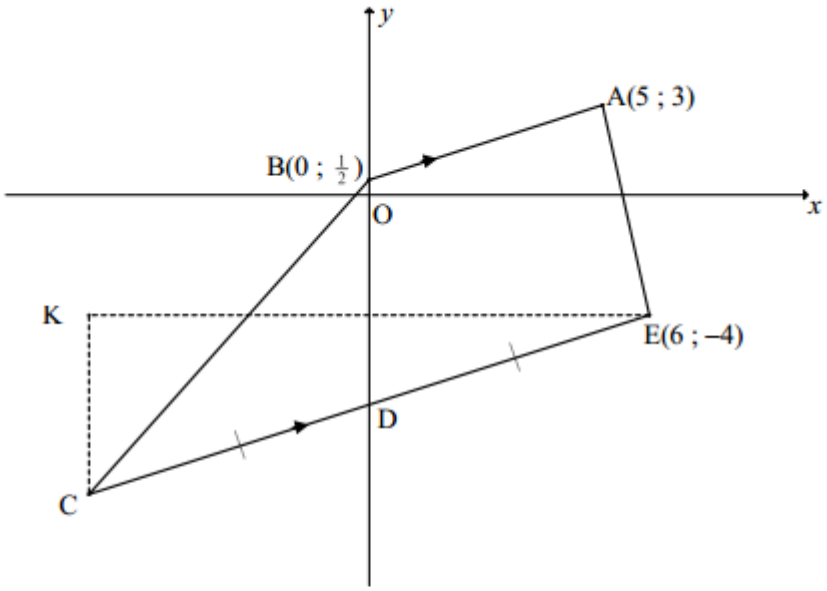
QUESTION/*VRAAG* 3

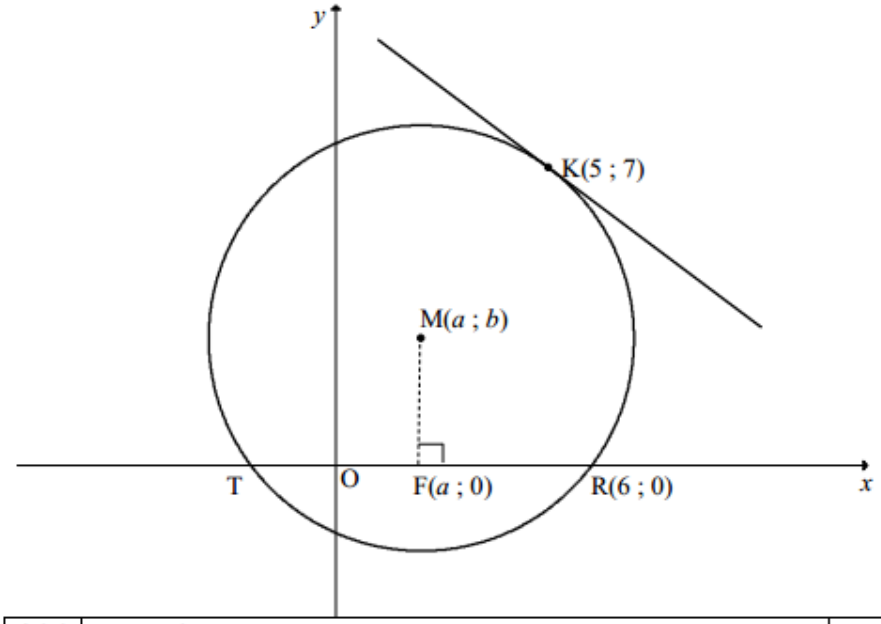


3.1	$m_{AB}=\frac{3-\frac{1}{2}}{5-0}$ $m_{AB}=\frac{1}{2}$ Answer only 2/2	✓ substitution ✓ answer (2)

3.2	$m_{CE} = m_{BA} = \frac{1}{2}$ $-4 = \frac{1}{2}(6) + c$ **OR/*OF*** $y-(-4) = \frac{1}{2}(x-6)$ $c = -7$ $y = \frac{1}{2}x - 7$	✓ gradient ✓ substitution of E ✓ answer (3)
3.3.1	D(0 ; –7) $\frac{x_C + 6}{2} = 0$ $\frac{y_C + (-4)}{2} = -7$ $x_C = -6$ $y_C = -10$ C(–6 ; –10) Answer only 3/3	✓ D(0 ; –7) ✓ $x_C = -6$ ✓ $y_C = -10$ (3)
3.3.2	Area $\Delta BCD = \frac{1}{2}(7,5)(6)$ $= 22,5$ Area $\Delta ABD = \frac{1}{2}(7,5)(5)$ $= 18,75$ Area ABCD $= 22,5 + 18,75 = 41,25$ units^2	✓ subst of correct base and height into the area formula ✓ area $\Delta BCD = 22,5$ ✓ area $\Delta ABD = 18,75$ ✓ answer (4)

3.4.1	K(–6 ; –4)	✓ $x_K = -6$ ✓ $y_K = -4$ (2)
3.4.2a	KC = 6 units; KE = 12 units; $CE = \sqrt{(6)^2 + (12)^2}$ [Pythagoras] $CE = \sqrt{180} = 6\sqrt{5} = 13,42$ Perimeter $\Delta KEC = 6 + 12 + \sqrt{180}$ $= 31,42$ units	✓ KC = 6 units ✓ KE = 12 units ✓ CE ✓ answer (4)
3.4.2b	$\tan K\hat{C}E = \frac{KE}{KC} = \frac{12}{6} = 2$ $K\hat{C}E = 63,43°$ **OR/*OF*** $\sin K\hat{C}E = \frac{KE}{CE} = \frac{12}{\sqrt{180}} = \frac{2\sqrt{5}}{5}$ $K\hat{C}E = 63,43°$ **OR/*OF*** $m_{CE} = \frac{1}{2}$ $\tan\theta = \frac{1}{2}$ $\theta = 26,57°$ $K\hat{C}E = 90° - 26,57°$ $K\hat{C}E = 63,43°$ **OR/*OF*** $KE^2 = KC^2 + CE^2 - 2(KC)(CE)\cos K\hat{C}E$ $(12)^2 = (6)^2 + \left(\sqrt{180}\right)^2 - 2(6)\left(\sqrt{180}\right)(\cos K\hat{C}E)$ $\cos K\hat{C}E = \frac{\sqrt{5}}{5}$ $K\hat{C}E = 63,43°$	✓ trig ratio ✓ $\tan K\hat{C}E = 2$ ✓ answer (3) ✓ trig ratio ✓ $\sin K\hat{C}E = \frac{12}{\sqrt{180}}$ ✓ answer (3) ✓ $\tan\theta = \frac{1}{2}$ ✓ $\theta = 26,57°$ ✓ answer (3) ✓ substitution into cosine rule ✓ trig ratio ✓ answer (3)

QUESTION/*VRAAG* 4



4.1.1	$y = x + 1$ $b = a + 1$	✓ $b = a + 1$ (1)
4.1.2	$MR^2 = MK^2$ $(a-6)^2 + (b-0)^2 = (a-5)^2 + (b-7)^2$ $(a-6)^2 + (a+1)^2 = (a-5)^2 + (a+1-7)^2$ $a^2 + 2a + 1 = a^2 - 10a + 25$ $12a = 24$ $a = 2$ $b = 3$ $\therefore$M(2 ; 3)	✓ equating radii / solving simultaneously ✓ substitution $b = a + 1$ ✓ $12a = 24$ ✓ $a = 2$ ✓ $b = 3$ (5)
4.2.1	$(6-2)^2 + (0-3)^2 = r^2$ $r = 5$ **OR/*OF*** $(2-5)^2 + (3-7)^2 = r^2$ $r = 5$ Answer only 2/2	✓ substitution R and M ✓ $r = 5$ (2) ✓ substitution K and M ✓ $r = 5$ (2)

4.2.2	T(–2 ; 0) TR = 8 units [line from centre ⊥ to chord] **OR/*OF*** M(2 ; 3) F(a ; 0) FR = 4 units TR = 8 units [line from centre ⊥ to chord] **OR/*OF*** $(x-2)^2+(0-3)^2=25$ $x^2-4x+4+9=25$ $x^2-4x-12=0$ $(x-6)(x+2)=0$ $x=6$ or $x=-2$ TR = 8 units Answer only 2/2	✓ T(–2 ; 0) ✓ answer (2) ✓ 4 units ✓ answer (2) ✓ x values ✓ answer (2)
4.3	$m_{\text{radius}}=\frac{7-3}{5-2}$ $m_{\text{radius}}=\frac{4}{3}$ $m_{\text{tangent}}=-\frac{3}{4}$ $7=-\frac{3}{4}(5)+c$ **OR/*OF*** $y-7=-\frac{3}{4}(x-5)$ $c=\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$ $y=-\frac{3}{4}x+\frac{43}{4}$	✓ substitution ✓ $m_{\text{radius}}=\frac{4}{3}$ ✓ $m_{\text{tangent}}=-\frac{3}{4}$ ✓ substitution ✓ answer (5)
4.4.1	N(2 ; –2)	✓ $x_N=2$ ✓ $y_N=-2$ (2)
4.4.2	$(-2-2)^2+(0+2)^2=r^2$ $r^2=20$ $(x-2)^2+(y+2)^2=20$	✓ substitution ✓ $r^2=20$ ✓ answer (3)
		[20]

QUESTION/*VRAAG* 5

5.1.1	$\sin 220°$ $=-\sin 40°$ $=-p$	✓ $-\sin 40°$ ✓ answer (2)
5.1.2	$\cos^2 50°$ $=\sin^2 40°$ $=p^2$	✓ $\sin^2 40$ ✓ answer (2)
5.1.3	$\cos(-80°)$ $=\cos 80°$ $=1-2\sin^2 40°$ $=1-2p^2$ **OR** $\cos(-80°)$ $=\cos 80°$ $=\cos(30°+50°)$ $=\cos 30°\cos 50°-\sin 30°\sin 50°$ $=\frac{\sqrt{3}p}{2}-\frac{\sqrt{1-p^2}}{2}$	✓ $\cos 80°$ ✓ double angle ✓ answer (3) ✓ $\cos 80°$ ✓ expansion ✓ answer (3)
5.2.1	LHS $=\tan x(1-\cos^2 x)+\cos^2 x$ $=\frac{\sin x}{\cos x}(\sin^2 x)+\cos^2 x$ $=\frac{\sin^3 x+\cos^3 x}{\cos x}$ $=\frac{(\sin x+\cos x)(\sin^2 x-\sin x\cos x+\cos^2 x)}{\cos x}$ $=\frac{(\sin x+\cos x)(1-\sin x\cos x)}{\cos x}$ $=$ RHS **OR**	✓ $\frac{\sin x}{\cos x}$ ✓ $\sin^2 x$ ✓ simplification ✓ factorisation of cubes ✓ $\sin^2 x+\cos^2 x=1$ (5)

	$\text{RHS} = \dfrac{(\sin x + \cos x)(1 - \sin x \cos x)}{\cos x}$ $= \dfrac{\sin x - \sin^2 x \cos x + \cos x - \sin x \cos^2 x}{\cos x}$ $= \tan x - \sin^2 x + 1 - \sin x \cos x$ $= \tan x + \cos^2 x - \sin x \cos x$ $= \tan x\left(1 - \dfrac{\sin x \cos x}{\tan x}\right) + \cos^2 x$ $= \tan x\left(1 - \dfrac{\sin x \cos x}{\frac{\sin x}{\cos x}}\right) + \cos^2 x$ $= \tan x\left(1 - \cos^2 x\right) + \cos^2 x$ $= \text{LHS}$	✓ multiplication ✓ ÷ by $\cos x$ ✓ $-\sin^2 x + 1 = \cos^2 x$ ✓ factorisation ✓ $\tan x = \dfrac{\sin x}{\cos x}$ (5)
5.2.2	$\cos x = 0$ or where $\tan x$ is undefined $x = 90° + k.360°$ or $x = 270° + k.360°$ $x = 90°$ or $x = -90°$	✓ $\cos x = 0$ or $\tan x$ undefined ✓ $x = 90°$ ✓ $x = -90°$ (3)
5.3.1	$\dfrac{\sin 150° + \cos^2 x - 1}{2}$ $= \dfrac{\sin 30° + \cos^2 x - 1}{2}$ $= \dfrac{\frac{1}{2} - \left(1 - \cos^2 x\right)}{2}$ $= \left(\dfrac{1}{2} - \sin^2 x\right) \times \dfrac{1}{2}$ $= \dfrac{1 - 2\sin^2 x}{4}$ $= \dfrac{\cos 2x}{4}$	✓ $\sin 30°$ ✓ $\sin 30° = \dfrac{1}{2}$ ✓ factor ✓ $1 - \cos^2 x = \sin^2 x$ ✓ simplification ✓ answer in terms of $\cos 2x$ (6)
5.3.2	$\dfrac{\sin 150° + \cos^2 x - 1}{2} = \dfrac{1}{25}$ $\dfrac{\cos 2x}{4} = \dfrac{1}{25}$ $\cos 2x = \dfrac{4}{25}$ $\text{ref}\angle = 80,79...°$ $2x = 80,79...° + k.360°$ or $2x = 279,20...° + k.360°$ $x = 40,40° + k.180°$ or $x = 139,60° + k.180°$; $k \in Z$	✓ answer 5.3.1 $= \dfrac{1}{25}$ ✓ $2x = 80,79°$ ✓ $2x = 279,20...°$ ✓ $x = 40,40°$ and $x = 139,60°$ ✓ $+k.180°$; $k \in Z$ (5)

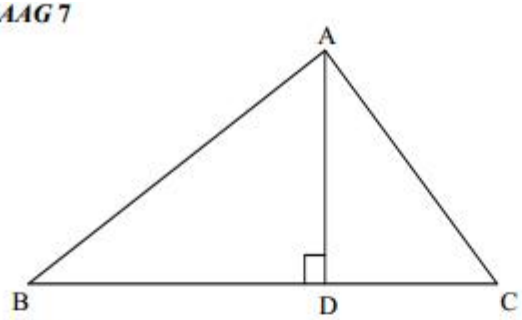
	OR $\frac{\sin 150° + \cos^2 x - 1}{2} = \frac{1}{25}$ $\sin 150° + \cos^2 x - 1 = \frac{2}{25}$ $\sin 30° + \cos^2 x - 1 = \frac{2}{25}$ $\cos^2 x = \frac{29}{50}$ $\cos x = \pm\sqrt{\frac{29}{50}}$ $x = 40,40° + k.360°$ or $x = 319,60° + k.360°$; $k \in Z$ or $x = 139,60° + k.360°$ or $x = 220,40° + k.360°$; $k \in Z$	✓ $\cos^2 x = \frac{29}{50}$ ✓ $x = 40,40°$ ✓ $x = 139,60°$ ✓ $x = 220,40°$ and $x = 319,60°$ ✓ $+ k.360°$; $k \in Z$ (5)
		[26]

QUESTION/*VRAAG* 6

6.1	Period = $360°$	✓ $360°$ (1)
6.2	Amplitude = 1	✓ 1 (1)
6.3	$a=-45°$	✓ $a=-45°$ (1)
6.4	$\sin 2x=k$ $k=\sin(2\times 165°)$ **OR** $k=\sin(2\times(-75°))$ $k=\sin 330°$ $k=\sin(-150°)$ $k=-\sin 30°$ $k=-\frac{1}{2}$ **OR** $k=\cos(165°-45°)$ **OR** $k=\cos(-75°-45°)$ $k=\cos 120°$ $k=\cos(-120°)$ $k=-\cos 60°$ $k=-\frac{1}{2}$	✓ $-\sin 30°$ ✓ $-\frac{1}{2}$ (2) ✓ $-\cos 60°$ ✓ $-\frac{1}{2}$ (2)
6.5	Points of intersection are translated $60°$ to the left $x=-15°$	✓ $x=-15°$ (1)
6.6	$\sqrt{2}\sin 2x=\sin x+\cos x$ $\sin 2x=\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x$ $\sin 2x=\sin 45°\sin x+\cos 45°\cos x$ $\sin 2x=\cos(45°-x)$ **OR** $\sin 2x=\cos(x-45°)$ $\therefore$ 2 roots in the interval $x\in[-90°;90°]$	✓ division by $\sqrt{2}$ ✓ special angles ✓ $\cos(45°-x)$ or $\cos(x-45°)$ ✓ answer (4)
		[10]

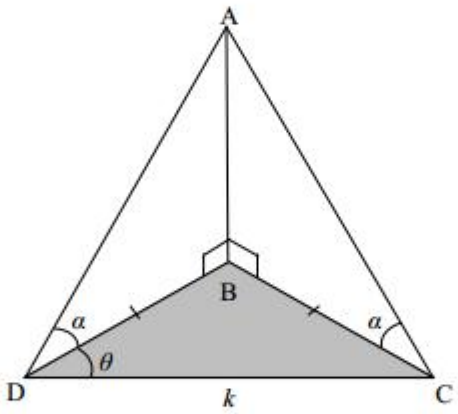
QUESTION/*VRAAG* 7

7.1



7.1.1	$\sin \hat{B} = \frac{AD}{AB}$ $AD = AB\sin \hat{B}$	✓ $\sin \hat{B} = \frac{AD}{AB}$ ✓ answer (2)
7.1.2	Area of $\Delta ABC = \frac{1}{2}(BC)(AD)$ $\therefore$ Area of $\Delta ABC = \frac{1}{2}(BC)(AB)\sin \hat{B}$	✓ $\frac{1}{2}(BC)(AD)$ (1)

7.2



7.2.1	In ΔADB $\sin\alpha = \frac{AB}{AD}$ $AD = \frac{AB}{\sin\alpha}$ In ΔABC $\sin\alpha = \frac{AB}{AC}$ $AC = \frac{AB}{\sin\alpha}$ $AD = AC$ **OR** In ΔADB and ΔACB	✓ $\sin\alpha = \frac{AB}{AD}$ ✓ $\sin\alpha = \frac{AB}{AC}$ (2)

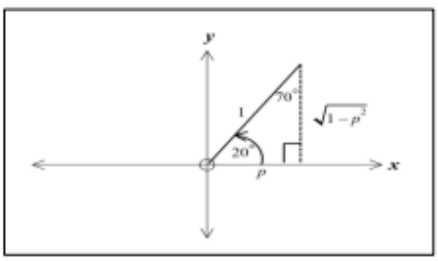
	$AB = AB$	[common side]	
	$A\hat{B}D = A\hat{B}C = 90°$	[given]	
	$BD = BC$	[given]	
	$\Delta ADB \equiv \Delta ACB$	[S∠S]	✓ $\Delta ADB \equiv \Delta ACB$ ✓ R
	$\therefore AD = AC$		
			(2)
	OR		
	In ΔADB and ΔACB		
	$A\hat{D}B = A\hat{C}B = \alpha$	[given]	
	$A\hat{B}D = A\hat{B}C = 90°$	[given]	
	$AB = AB$ **OR** $BD = BC$	[common side **OR** given]	
	$\therefore \Delta ADB \equiv \Delta ACB$	[∠∠S]	✓ $\Delta ADB \equiv \Delta ACB$ ✓ R
	$\therefore AD = AC$		
			(2)
	OR		
	$AD^2 = AB^2 + DB^2$	[Pythagoras]	✓ both Pythagoras statements
	$AC^2 = AB^2 + BC^2$	[Pythagoras]	
	But $DB = BC$	[given]	✓ $DB = BC$
	$\therefore AD^2 = AC^2$		
	$\therefore AD = AC$		
			(2)
7.2.2	$\frac{BD}{\sin\theta} = \frac{k}{\sin(180° - 2\theta)}$		✓ substitution of $(180° - 2\theta)$ into sine rule
	$BD = \frac{k\sin\theta}{\sin 2\theta}$		✓ reduction
	$BD = \frac{k\sin\theta}{2\sin\theta\cos\theta}$		✓ double angle
	$BD = \frac{k}{2\cos\theta}$		
			(3)
	OR		
	$BC^2 = k^2 + BD^2 - 2k(BD)\cos\theta$		✓ substitution into cosine-rule
	$BD^2 = k^2 + BD^2 - 2k(BD)\cos\theta$		✓ substitution BC with BD into cosine-rule
	$k^2 - 2k(BD)\cos\theta = 0$		
	$2k(BD)\cos\theta = k^2$		✓ simplification in terms of BD
	$\therefore BD = \frac{k}{2\cos\theta}$		
			(3)

7.2.3	Area of $\Delta BCD = \frac{1}{2}(DC)(BD)(\sin C\hat{D}B)$ $= \frac{1}{2}k\left(\frac{k}{2\cos\theta}\right)\sin\theta$ $= \frac{1}{4}k^2\tan\theta$ **OR** Area of $\Delta BCD = \frac{1}{2}(BD)(BC)(\sin(180°-2\theta))$ $= \frac{1}{2}\left(\frac{k}{2\cos\theta}\right)\left(\frac{k}{2\cos\theta}\right)(\sin 2\theta)$ $= \frac{2k^2\sin\theta\cos\theta}{8\cos\theta\cos\theta}$ $= \frac{1}{4}k^2\tan\theta$	✓ substitution into area rule ✓ $\frac{\sin\theta}{\cos\theta} = \tan\theta$ ✓ $\frac{1}{4}k^2\tan\theta$ (3) ✓ substitution into area rule ✓ $\frac{\sin\theta}{\cos\theta} = \tan\theta$ ✓ $\frac{1}{4}k^2\tan\theta$ (3)
		[11]

May/June 2023

QUESTION/*VRAAG 5*

5.1	$\frac{1-\sin(-\theta)\cos(90°+\theta)}{\cos(\theta-360°)}$ $= \frac{1-(-\sin\theta)(-\sin\theta)}{\cos\theta}$ $= \frac{1-\sin^2\theta}{\cos\theta}$ $= \frac{\cos^2\theta}{\cos\theta}$ $= \cos\theta$	✓ $-\sin\theta$ ✓ $-\sin\theta$ ✓ $\cos\theta$ ✓ $\cos^2\theta$ ✓ answer (5)
5.2.1	$\cos 200°$ $= -\cos 20°$ $= -p$	✓ reduction ✓ answer (2)
5.2.2	$\sin(-70°)$ $= -\sin 70°$ $= -\cos 20°$ $= -p$ **OR/*OF*** $\sin(-70°)$ $= -\sin 70°$ $= -p$	✓ reduction ✓ answer (2) ✓ reduction ✓ answer (2)

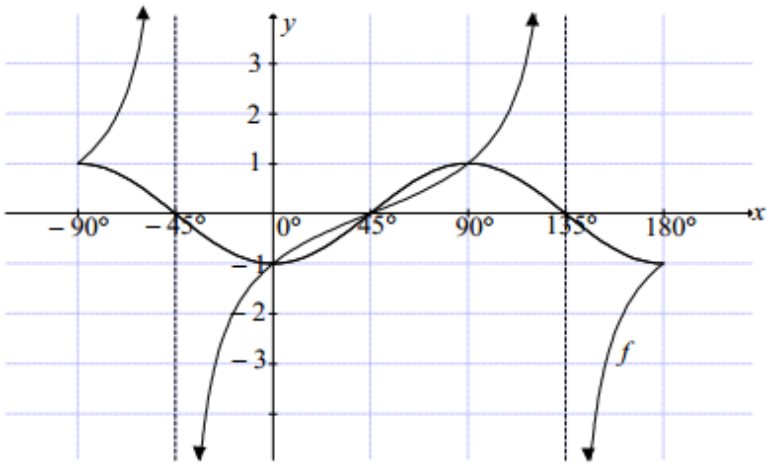


5.2.3	$\sin 10°$ $\cos(2(10°)) = 1 - 2\sin^2 10°$ $2\sin^2 10° = 1 - \cos 20°$ $\sin 10° = \sqrt{\frac{1-\cos 20°}{2}}$ $\sin 10° = \sqrt{\frac{1-p}{2}}$ **OR/*OF*** $\sin 10°$ $\sin(30° - 20°)$ $= \sin 30°\cos 20° - \cos 30°\sin 20°$ $= \frac{1}{2}p - \frac{\sqrt{3}}{2}\sqrt{1-p^2} = \frac{p - \sqrt{3}\sqrt{1-p^2}}{2}$ **OR/*OF***	✓ double angle ✓ $\sin 10°$ as subject ✓ answer (3) ✓ using special angle ✓ expanding ✓ answer (3)
	$\sin 10°$ $\sin(70° - 60°)$ $= \sin 70°\cos 60° - \cos 70°\sin 60°$ $= p.\frac{1}{2} - \sqrt{1-p^2} \times \frac{\sqrt{3}}{2} = \frac{p - \sqrt{3}\sqrt{1-p^2}}{2}$ **OR/*OF*** $\sin 10°$ $= \cos 80°$ $\cos(60° + 20°)$ $= \cos 60°\cos 20° - \sin 60°\sin 20°$ $= \frac{1}{2}p - \frac{\sqrt{3}}{2}.\sqrt{1-p^2}$	✓ using special angle ✓ expanding ✓ answer (3) ✓ using special angle ✓ expanding ✓ answer (3)

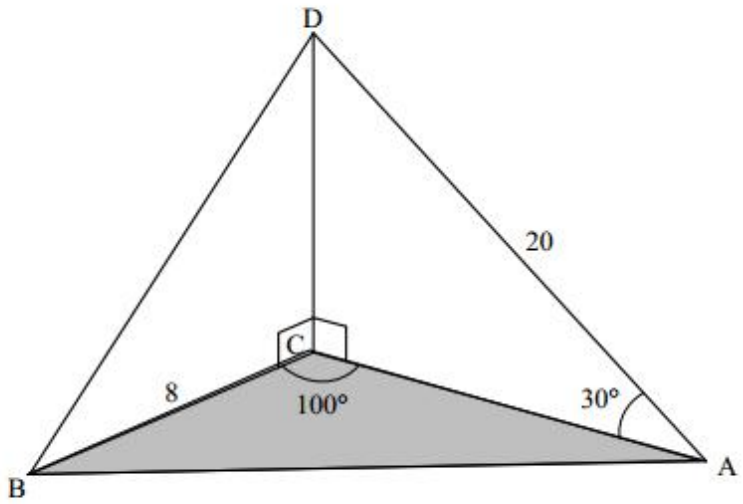
5.3	$\cos(A+55°)\cos(A+10°)+\sin(A+55°)\sin(A+10°)$ $=\cos[A+55°-(A+10°)]$ $=\cos 45°$ $=\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$	✓✓ compound identity ✓ answer (3)
5.4.1	LHS $=\frac{\cos 2x+\sin 2x-\cos^2 x}{\sin x-2\cos x}$ RHS $=-\sin x$ $=\frac{\cos^2 x-\sin^2 x+2\sin x\cos x-\cos^2 x}{\sin x-2\cos x}$ $=\frac{-\sin^2 x+2\sin x\cos x}{\sin x-2\cos x}$ $=\frac{-\sin x(\sin x-2\cos x)}{\sin x-2\cos x}$ $=-\sin x$ $\therefore$ LHS = RHS	✓ $\cos^2 x-\sin^2 x$ ✓ $2\sin x\cos x$ ✓ common factor of $-\sin x$ (3)
5.4.2	$\frac{\cos 2x+\sin 2x-\cos^2 x}{-3\sin^2 x+6\sin x\cos x}$ $=\frac{\cos 2x+\sin 2x-\cos^2 x}{-3\sin x(\sin x-2\cos x)}$ $=\frac{\cos 2x+\sin 2x-\cos^2 x}{(\sin x-2\cos x)}\times\frac{1}{-3\sin x}$ $=(-\sin x)\times\frac{1}{-3\sin x}$ $=\frac{1}{3}$	✓ common factor of $-3\sin x$ ✓ substitution ✓ answer (3)

5.5.1	$3\tan 4x=-2\cos 4x$ $3\left(\frac{\sin 4x}{\cos 4x}\right)=-2\cos 4x$ $3\sin 4x+2\cos^2 4x=0$ $3\sin 4x+2(1-\sin^2 4x)=0$ $-2\sin^2 4x+3\sin 4x+2=0$ $2\sin^2 4x-3\sin 4x-2=0$ $(2\sin 4x+1)(\sin 4x-2)=0$ $\sin 4x=-\frac{1}{2}$ or $\sin 4x\neq 2$	✓ identity ✓ $1-\sin^2 4x$ ✓ standard form ✓ factors (4)
5.5.2	$\sin 4x=-\frac{1}{2}$ *ref.* $\angle=30°$ $4x=210°+k.360°$ or $4x=330°+k.360°$ $x=52{,}5°+k.90°$; $k\in Z$ $x=82{,}5°+k.90°$; $k\in Z$	✓ 210° ; 330° ✓ 52,5° ; 82,5° ✓ $k.90°$; $k\in Z$ (3)
		[28]

QUESTION/VRAAG 6

6.1	Period = 180°	✓ answer (1)
6.2		✓ x-intercepts ✓ turning points ✓ end points (3)
6.3	$y \in [-1;1]$ **OR/*OF*** $-1 \le y \le 1$	✓ answer (1)
6.4	$g(x) = -\cos 2x$ $g(x+45°) = -\cos 2(x+45°)$ $= -\cos(2x+90°)$ $= \sin 2x$	✓ $-\cos 2(x+45°)$ ✓ answer (2)
6.5.1	$x \in (-90°;-45°)$ **OR/*OF*** $-90° < x < -45°$	✓✓ $x \in (-90°;-45°)$ (2)
6.5.2	$2\cos 2x - 1 > 0$ $\cos 2x > \frac{1}{2}$ $-\cos 2x < -\frac{1}{2}$ $x \in (-30°;30°)$ **OR/*OF*** $-30° < x < 30°$	✓ $\cos 2x > \frac{1}{2}$ ✓ $-\cos 2x < -\frac{1}{2}$ ✓ $x = \pm 30°$ ✓ interval (4)
		[13]

QUESTION/*VRAAG 7*



7.1.1	$\frac{AC}{20} = \cos 30°$ $AC = 20\cos 30°$ $AC = 10\sqrt{3} = 17,32$ units **OR/*OF*** $\frac{AC}{\sin 60°} = \frac{20}{\sin 90°}$ $\therefore AC = 20\sin 60 = 17,32$	✓ trig ratio ✓ answer (2) ✓ trig ratio ✓ answer (2)
7.1.2	$AB^2 = AC^2 + BC^2 - 2AC.BC\cos A\hat{C}B$ $AB^2 = \left(10\sqrt{3}\right)^2 + 8^2 - 2\left(10\sqrt{3}\right)(8)\cos 100°$ $AB = 20,30$ units	✓ cosine formula ✓ substitution into cosine formula ✓ answer (3)
7.2	$\frac{\sin A\hat{D}B}{AB} = \frac{\sin A\hat{B}D}{AD}$ $\frac{\sin A\hat{D}B}{20,3} = \frac{\sin 73,4°}{20}$ $\sin A\hat{D}B = \frac{20,3\sin 73,4°}{20}$ $A\hat{D}B = 76,58°$	✓ sine formula in ΔABD ✓ substitution into sine formula ✓ answer (3)

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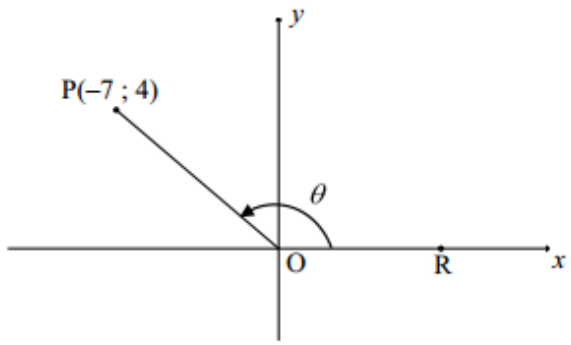
QUESTION/*VRAAG* 5



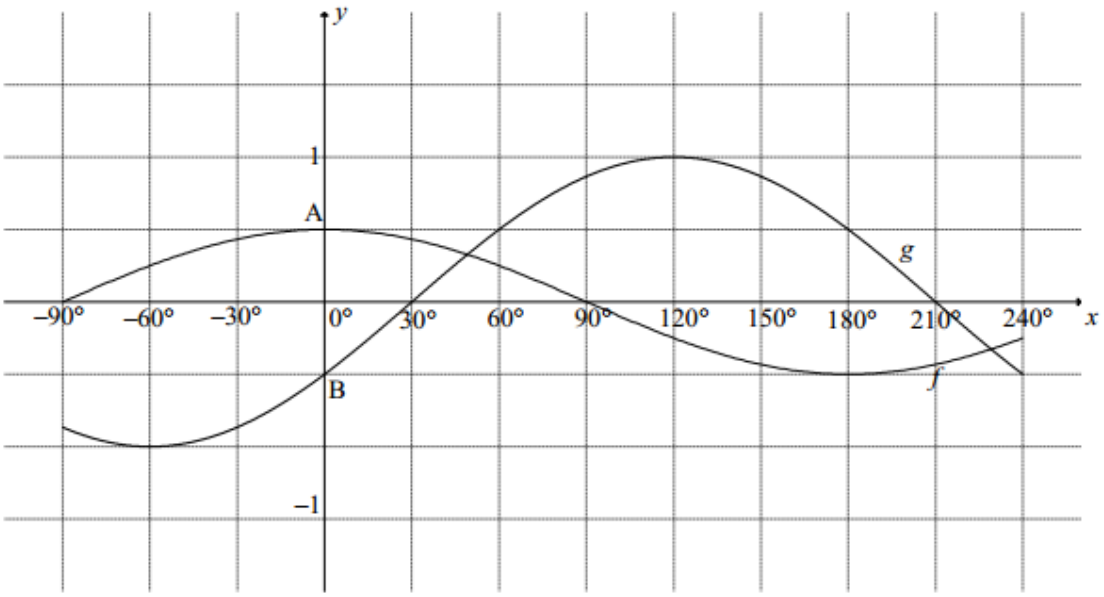
5.1.1	$OP=\sqrt{(-7)^2+(4)^2}$ $=\sqrt{65}$ Answer only 2/2	✓ substitution ✓ answer (2)
5.1.2(a)	$\tan\theta=\frac{4}{-7}$	✓ answer (1)
5.1.2(b)	$\cos(\theta-180°) = -\cos\theta$ $= \frac{7}{\sqrt{65}}$	✓ reduction ✓ answer (2)
5.2	$\sin x\cos x+\sin x=3\cos^2 x+3\cos x$ $\sin x\cos x+\sin x-3\cos^2 x-3\cos x=0$ $\sin x(\cos x+1)-3\cos x(\cos x+1)=0$ $(\cos x+1)(\sin x-3\cos x)=0$ $\cos x=-1$ or $\sin x=3\cos x$ $\tan x=3$ $x=180°+k.360°$ or $x=71{,}57°+k.180°$; $k\in Z$ **OR/*OF***	✓ RHS = 0 ✓ grouping ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=71{,}57°$ ✓ $+k.180°$; $k\in Z$ (7)
	$\sin x\cos x+\sin x=3\cos^2 x+3\cos x$ $\sin x\cos x+\sin x-3\cos^2 x-3\cos x=0$ $\sin x(\cos x+1)-3\cos x(\cos x+1)=0$ $(\cos x+1)(\sin x-3\cos x)=0$ $\cos x=-1$ or $\sin x=3\cos x$ $\tan x=3$ $x=180°+k.360°$ or $x=71{,}57°+k.360°$ or $x=251{,}57°+k.360°$; $k\in Z$	✓ RHS = 0 ✓ grouping ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=71{,}57°$ and $251{,}57°$ ✓ $+k.360°$; $k\in Z$ (7)

5.3.1	$\text{LHS} = \frac{\sin 3x}{1-\cos 3x} \times \frac{1+\cos 3x}{1+\cos 3x}$ $= \frac{(\sin 3x)(1+\cos 3x)}{(1-\cos 3x)(1+\cos 3x)}$ $= \frac{(\sin 3x)(1+\cos 3x)}{1-\cos^2 3x}$ $= \frac{(\sin 3x)(1+\cos 3x)}{\sin^2 3x}$ $= \frac{1+\cos 3x}{\sin 3x}$ $= \text{RHS}$ **OR/*OF*** $\text{LHS} = \frac{\sin 3x}{1-\cos 3x} \times \frac{\sin 3x}{\sin 3x}$ $= \frac{\sin^2 3x}{\sin 3x(1-\cos 3x)}$ $= \frac{1-\cos^2 3x}{\sin 3x(1-\cos 3x)}$ $= \frac{(1-\cos 3x)(1+\cos 3x)}{\sin 3x(1-\cos 3x)}$ $= \frac{1+\cos 3x}{\sin 3x}$ $= \text{RHS}$	✓ multiply by "1" ✓ $1-\cos^2 3x$ ✓ square identity (3) ✓ multiply by "1" ✓ square identity ✓ factors (3)
5.3.2	undefined when $\sin 3x = 0$ and $1-\cos 3x = 0$ $3x = 0°$ or $3x = 180°$ and $3x = 0°$ or $3x = 360°$ $x = 0°$ or $x = 60°$	✓ $\sin 3x = 0$ and $1-\cos 3x = 0$ ✓ $0°$ ✓ $60°$ (3)
		[18]

QUESTION/*VRAAG* 6

6.1	$\frac{\sin 10°}{\cos 440°} + \tan(360° - \theta).\sin 2\theta$	
	$= \frac{\cos 80°}{\cos 80°} - \tan\theta(2\sin\theta\cos\theta)$	✓ $-\tan\theta$ ✓ $\cos 80°$ ✓ co-ratio ✓ double angle
	$= 1 - \frac{\sin\theta}{\cos\theta}(2\sin\theta\cos\theta)$	✓ quotient identity
	$= 1 - 2\sin^2\theta$	
	$= \cos 2\theta$	✓ answer (6)
6.2.1	$\sin(60° + 2x) + \sin(60° - 2x) = k\cos 2x$	
	$(\sin 60°\cos 2x + \cos 60°\sin 2x) + (\sin 60°\cos 2x - \cos 60°\sin 2x) = k\cos 2x$ $2\sin 60°\cos 2x = k\cos 2x$	✓ both expansions correct
	$2\left(\frac{\sqrt{3}}{2}\right)\cos 2x = k\cos 2x$	✓ special ∠s
	$\therefore k = \sqrt{3}$	✓ answer (3)
6.2.2	$\tan 60°[\sin(60° + 2x) + \sin(60° - 2x)]$	
	$= \tan 60°[k\cos 2x]$	
	$= \sqrt{3}(\sqrt{3}\cos 2x)$	✓ special ∠
	$= 3(2\cos^2 x - 1)$	✓ double ∠s
	$= 3\left(2\left(\sqrt{t}\right)^2 - 1\right)$	✓ answer i.t.o t
	$= 6\left(\sqrt{t}\right)^2 - 3$	(3)
	$= 6t - 3$	
		[12]

QUESTION/*VRAAG* 7



7.1	$A\left(0;\frac{1}{2}\right)$ $B\left(0;-\frac{1}{2}\right)$ $AB=\frac{1}{2}-\left(-\frac{1}{2}\right)$ $=1$ unit Answer only 2/2	✓ y-values ✓ answer (2)
7.2	Range of f: $y\in\left[-\frac{1}{2};\frac{1}{2}\right]$ Range of $3f(x)+2$: $y\in\left[\frac{1}{2};3\frac{1}{2}\right]$ **OR/*OF*** $\frac{1}{2}\le y\le 3\frac{1}{2}$	✓ critical values ✓ answer (2)
7.3	$x=90°$	✓✓ $x=90°$ (2)
7.4.1	$x\in(30°;90°)\cup(210°;240°]$ **OR/*OF*** $30°<x<90°$ or $210°<x\le 240°$	✓ $x\in(30°;90°)$ ✓ $(210°;240°]$ (2) ✓ $30°<x<90°$ ✓ $210°<x\le 240°$ (2)
7.4.2	$x\in(-55°;125°)$ **OR/*OF*** $-55°<x<125°$	✓ critical values ✓ answer (2) ✓ critical values ✓ answer (2)
		[10]

QUESTION/*VRAAG* 8

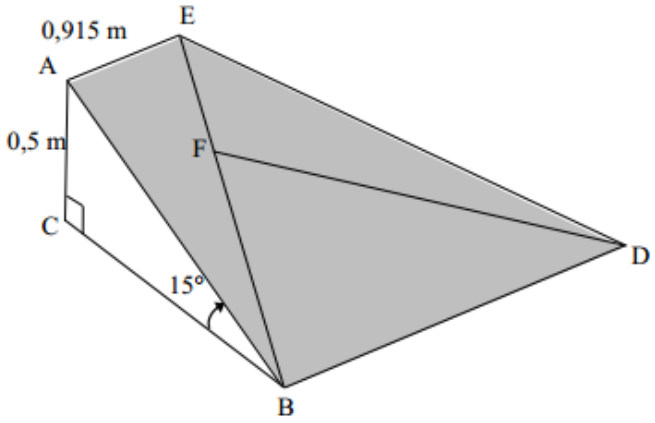


FIGURE I

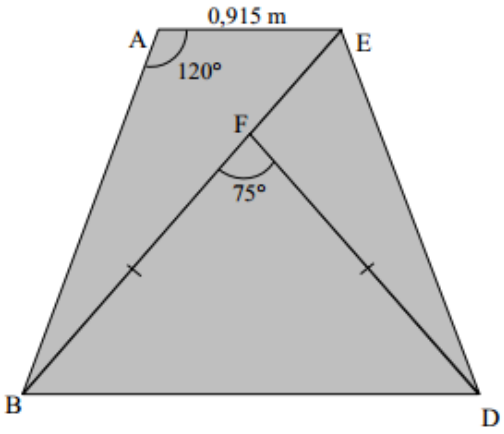


FIGURE II (top view)

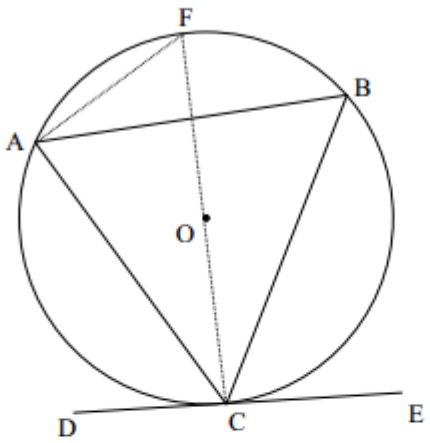
8.1	$\frac{0,5}{AB} = \sin 15°$ $AB = \frac{0,5}{\sin 15°}$ $AB = 1,93$ m Answer only 2/2	✓ trig ratio ✓ answer (2)
8.2	$BE^2 = AB^2 + AE^2 - 2(AB)(AE)\cos B\hat{A}E$ $BE^2 = (1,93)^2 + (0,915)^2 - 2(1,93)(0,915)(\cos 120°)$ $BE = 2,52$ m	✓ correct use of cosine rule ✓ substitution ✓ answer (3)
8.3	$BF = FD = \frac{5}{7}(2,52) = 1,80$ m Area $\Delta BFD = \frac{1}{2}(BF)(FD)\sin B\hat{F}D$ $= \frac{1}{2}(1,8)(1,8)(\sin 75°)$ $= 1,56\ m^2$	✓ BF ✓ correct substitution into the area rule ✓ answer (3)
		[8]

Euclidean Geometry

May/June 2024

QUESTION/*VRAAG* 8

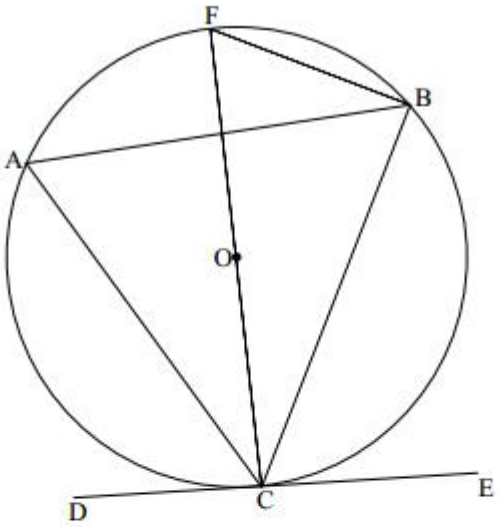
8.1



	Construction: Draw diameter CF and draw AF *Konstruksie: Trek middellyn* CF *en verbind* AF		✓Constr
	$F\hat{C}E = 90°$	[tan ⊥ radius/*raaklyn ⊥ radius*]	✓ S ✓ R
	$F\hat{A}C = 90°$	[∠ in semi circle/*∠ in halwe sirkel*]	✓ S/R
	$F\hat{A}B = F\hat{C}B$	[∠s same segment/*∠e dieselfde segm*]	✓ S/R
	$\therefore B\hat{A}C = B\hat{C}E$ $\therefore B\hat{C}E = \hat{A}$		(5)

OR

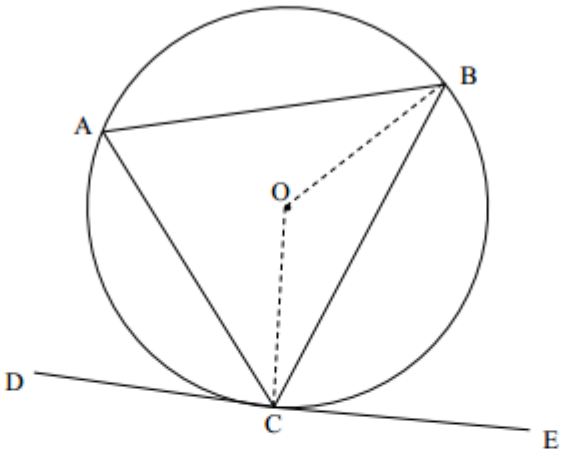
8.1



	Construction: Draw diameter CF and draw FB *Konstruksie: Trek middellyn* CF *en verbind* FB		✓ construction
	$F\hat{B}C = 90°$	[∠ in semi circle/*∠ in halwe sirkel*]	✓ S / R
	$B\hat{F}C + F\hat{C}B = 90°$	[sum of ∠s in Δ/*binne ∠e v Δ*]	
	$O\hat{C}E = 90°$	[tan ⊥ radius/ *raaklyn ⊥ radius*]	✓ S ✓ R
	$\therefore B\hat{C}E = \hat{F}$		
	but $\hat{A} = \hat{F}$	[∠s in same seg/*∠ in dies. segment*]	✓ S / R
	$\therefore B\hat{C}E = \hat{A}$		(5)

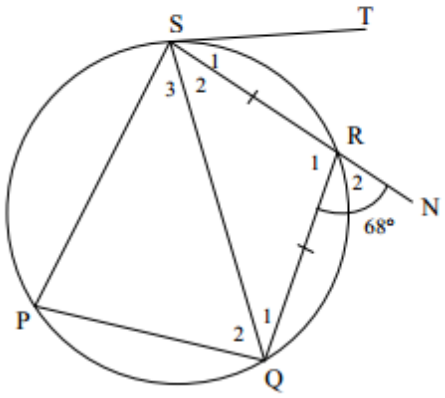
OR

8.1



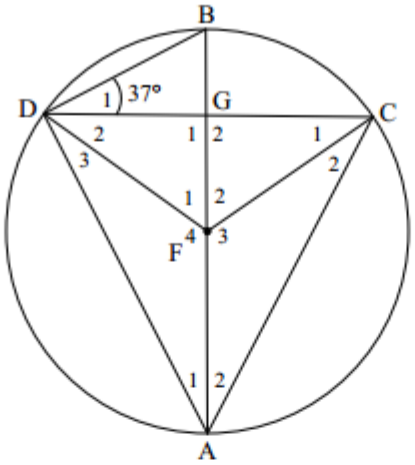
	Construction: Draw radii BO and OC *Konstruksie: Trek radiusse BO en OC*	✓ construction
	$O\hat{C}E = 90°$ or $B\hat{C}E = 90° - O\hat{C}B$ [tan ⊥ radius / *raaklyn ⊥ radius*]	✓ S ✓R
	$O\hat{C}B = O\hat{B}C$ [∠s opp equal sides/ *∠e teenoor gelyke sye*] $\therefore C\hat{O}B = 180° - 2O\hat{C}B$ [∠s of Δ/*∠e van Δ*]	✓ S
	$C\hat{A}B = 90° - O\hat{C}B$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore B\hat{C}E = C\hat{A}B$	✓ S/R (5)

8.2



8.2.1	$\hat{P} = \hat{R}_2 = 68°$	[ext ∠ of cyclic quad /*buite ∠ van kvh*]	✓ S ✓ R (2)
8.2.2	$\hat{Q}_1 = \hat{S}_2$ $\hat{Q}_1 + \hat{S}_2 = 68°$ $\therefore \hat{Q}_1 = 34°$	[∠s opp equal sides /*∠e teenoor gelyke sye*] [ext ∠ of Δ/ *buite ∠ van* Δ]	✓ S ✓ S (2)
8.2.3	$\hat{S}_1 = \hat{Q}_1 = 34°$	[tan-chord theorem/*∠ tussen rkl en koord*]	✓ S ✓ R (2)
			[11]

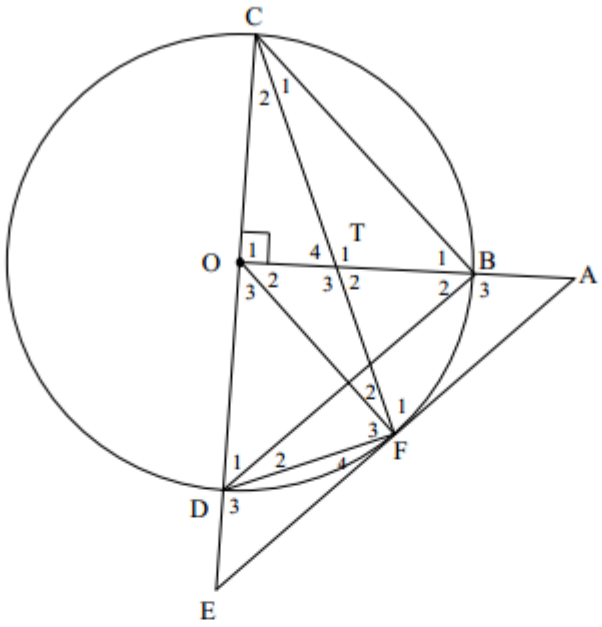
QUESTION/*VRAAG* 9



9.1	$\hat{A}_2 = \hat{D}_1 = 37°$	[∠s in the same seg/*∠e in dies segment*]	✓ S ✓ R
	$\hat{A}_1 = \hat{A}_2 = 37°$	[BA bisects CÂD/BA *halveer* CÂD]	
	$\hat{D}_3 = \hat{A}_1 = 37°$	[∠s opp equal sides/*∠e teenoor gelyke sye*]	✓✓ any other two statements
	$\hat{C}_2 = \hat{A}_2 = 37°$	[∠s opp equal sides/*∠e teenoor gelyke sye*]	(4)
9.2	$A\hat{D}G = 53°$	[∠ in semi circle/*∠ in halwe sirkel*]	✓ S ✓ R
	$\hat{A}_1 = 37°$	[proved in 9.1/*reeds bewys in 9.1*]	
	$\therefore \hat{G}_1 = 90°$	[sum of ∠s in Δ/*binne ∠e van* Δ]	✓ S
	$\therefore CG = DG$	[line from centre ⊥ to chord/ *lyn uit midpt. ⊥ op koord*]	✓ R (4)
	OR		
	$\hat{F}_2 = 2\hat{D}_1 = 74°$	[∠ at centre = 2 ×∠ at circumference/ *midpt. ∠s = 2×omtreks∠*]	✓ S ✓ R
	$\hat{D}_3 = 37°$	[proved in 9.1/*reeds bewys in 9.1*]	
	$\therefore \hat{D}_2 = 16°$	[∠ in semi circle/*∠ in halwe sirkel*]	
	$\hat{C}_1 = \hat{D}_2 = 16°$	[∠s opp equal sides/*∠e teenoor gelyke sye*]	
	$\therefore \hat{G}_2 = 90°$	[sum of ∠s in Δ/*binne ∠e van* Δ]	✓ S
	$\therefore CG = DG$	[line from centre ⊥ to chord/ *lyn uit midpt. ⊥ op koord*]	✓ R (4)

9.3	$\hat{F}_2 = 2\hat{D}_1 = 74°$ **OR** $\hat{F}_2 = 2\hat{A}_2 = 74°$	[∠ at centre = 2 ×∠ at circum./ *midpt. ∠s = 2×omtreks∠*]	✓ S
	$\frac{FG}{20} = \cos 74°$		✓ trig ratio
	$FG = 5,51$		✓ FG
	$\therefore BG = 14,49$ units		✓ answer (4)
	OR		
	$\hat{F}_2 = 2\hat{D}_1 = 74°$	[∠ at centre = 2 ×∠ at circumference *midpt. ∠ = 2×omtreks∠*]	✓ S
	$\frac{FG}{20} = \sin 16°$		✓ trig ratio
	$FG = 5,51$		✓ FG
	$\therefore BG = 14,49$ units		✓ answer (4)
	OR		
	$\frac{DG}{20} = \cos 16°$		✓ trig ratio
	$DG = 19,23$		✓ length of DG
	$\frac{BG}{19,23} = \tan 37°$		✓ trig ratio
	$BG = 14,49$ units		✓ answer (4)
	OR		
	$\frac{DG}{20} = \cos 16°$		✓ trig ratio
	$DG = 19,23$		✓ length of DG
	$FG^2 = FD^2 - DG^2$	[Pythagoras]	
	$FG^2 = 20^2 - (19,23)^2$		✓ correct use of Pythagoras
	$FG = 5,51$		
	$BG = 20 - 5,51$		
	$= 14,49$ units		✓ answer (4)
			[12]

QUESTION/*VRAAG* 10



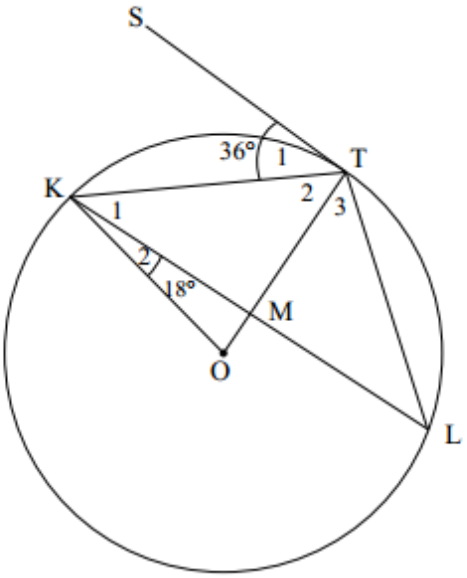
10.1	$\hat{O}_1 = 90°$	[given/*gegee*]	
	$\hat{F}_2 + \hat{F}_3 = 90°$	[$\angle$ in semi circle/$\angle$ *in halwe sirkel*]	✓ S ✓ R
	$\hat{O}_1 = \hat{F}_2 + \hat{F}_3 = 90°$		✓ S
	$\therefore$ TODF is a cyclic quad	[ext $\angle$ = int opp $\angle$/ *buite* $\angle$ = *teenoorst. binne* $\angle$] **OR** [converse ext $\angle$ of cyclic quad/ *omgekeerde buite* $\angle$ *v kvh*]	✓ R (4)
10.2	$\hat{T}_1 = \hat{T}_3$	[vert opp $\angle$s =/ *regoorstaande* $\angle$*e*]	✓ S / R
	But $\hat{D}_3 = \hat{T}_3$	[ext $\angle$ of cyclic quad/ *buite* $\angle$ *v kvh*]	✓ S ✓ R
	$\therefore \hat{T}_1 = \hat{D}_3$		(3)
10.3	In ΔDFE and ΔTFO		
	1) $\hat{D}_3 = \hat{T}_3$	[ext $\angle$ of cyclic quad/ *buite* $\angle$ *v kvh*]	✓ S
	2) $\hat{F}_4 = \hat{C}_2$	[tan-chord theorem/ $\angle$ *tussen rkl en koord*]	✓ S / R
	but $\hat{C}_2 = \hat{F}_2$	[$\angle$s opp equal sides/ $\angle$*e teenoor gelyke sye*]	✓ S
	$\therefore \hat{F}_4 = \hat{F}_2$		✓ S
	3) $\hat{E} = \hat{O}_2$	[3rd $\angle$ of Δ/$\angle$*e van* Δ]	✓ S **OR** R
	ΔTFO ||| ΔDFE	[$\angle\angle\angle$]	(5)

	OR		
	In ΔDFE and ΔTFO		
	1) $\hat{D}_3 = \hat{T}_3$	[ext $\angle$ of cyclic quad/*buite* $\angle$ *van* Δ]	✓ S
	2) $\hat{F}_4 = \hat{C}_2$	[tan-chord theorem/$\angle$ *tussen rkl en koord*]	✓ S / R
	$\hat{F}_2 + \hat{F}_3 = 90°$	[$\angle$ in semi circle/$\angle$ *in halwe sirkel*]	
	$\hat{D}_1 + \hat{D}_2 = 90° - \hat{C}_2$	[sum of $\angle$s in Δ/ *binne* $\angle$*e van* Δ]	
	$\hat{E} = 90° - 2\hat{F}_4$	[ext $\angle$ of Δ/ *buite* $\angle$ *van* Δ]	✓ S
	$\hat{O}_3 = 2\hat{C}_2$	[$\angle$ at centre = 2 ×$\angle$ at circumference/ *midpt.* $\angle$*s* = 2×*omtreks*$\angle$]	
	$\hat{O}_2 = 90° - 2\hat{F}_4$	[$\angle$s on a str line/$\angle$*e op 'n reguitlyn*]	✓ S
	$\hat{O}_2 = \hat{E}$		
	3) $\therefore \hat{F}_4 = \hat{F}_2$	[3rd $\angle$ of Δ/$\angle$*e van* Δ]	✓ S **OR** R
	$\Delta TFO \,|||\, \Delta DFE$	[$\angle\angle\angle$]	(5)
10.4	$\hat{B}_2 = \hat{D}_1$	[$\angle$s opp equal sides/$\angle$*e teenoor gelyke sye*]	✓ S / R
	$\hat{B}_2 = \hat{E}$	[given/*gegee*]	
	$\therefore \hat{D}_1 = \hat{E}$		
	$\therefore DB \parallel EA$	[corresp $\angle$s =/*ooreenkomstige* $\angle$*e gelyk*]	✓ R
			(2)
10.5	In ΔOEA		
	$DB \parallel EA$	[proven/*reeds bewys*]	
	$\frac{OD}{DE} = \frac{OB}{BA}$	[line $\parallel$ one side of Δ/*lyn* $\parallel$ *een sy van* Δ]	✓ R
		OR [prop theorem; DB $\parallel$ EA/ *eweredigheid stelling; DB* $\parallel$ *EA*]	
	$\therefore DE = \frac{DO.AB}{OB}$		✓ S
	$\frac{FO}{FE} = \frac{TO}{DE}$	[$\Delta TFO \,|||\, \Delta DFE$]	✓ S / R
	$DE = \frac{TO.FE}{FO}$		✓ S
	$\therefore \frac{DO.AB}{OB} = \frac{TO.FE}{FO}$		✓ S
	$\therefore \frac{DO.AB}{DO} = \frac{TO.FE}{DO}$	[DO = OB = FO]	
	$\therefore DO = \frac{TO.FE}{AB}$		(5)
			[19]

May/June 2023

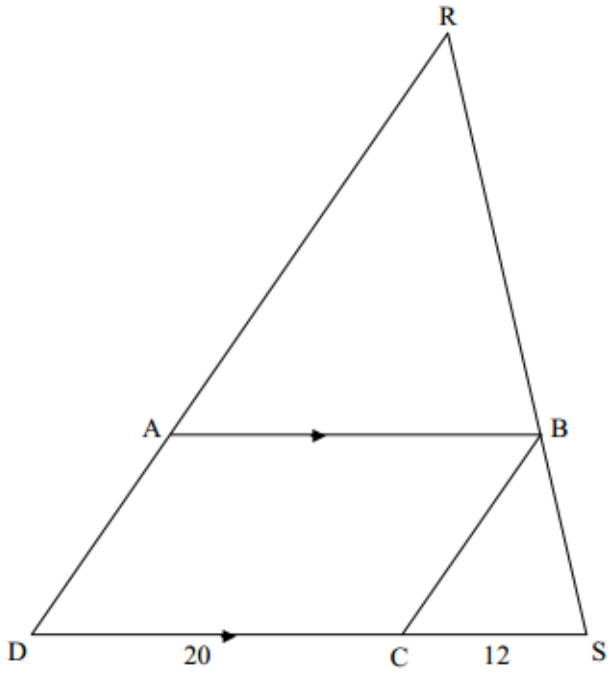
QUESTION/*VRAAG 8*

8.1



8.1.1(a)	$\hat{T}_2 = 54°$	[tan ⊥ rad]	✓ S ✓R (2)
8.1.1(b)	$\hat{L} = 36°$	[tan - chord theorem]	✓ S ✓R (2)
8.1.1(c)	$K\hat{O}T = 72°$	[∠ at centre = 2×∠ at circumference]	✓ S ✓R (2)
	OR/*OF*		
	$O\hat{K}T = \hat{T}_2 = 54°$	[∠s opposite = radii]	✓ S/R
	$K\hat{O}T = 180° - (54° + 54°)$ $= 72°$	[sum of int ∠'s of Δ]	✓ S (2)
8.1.2	$K\hat{M}O = 180° - (18° + 72°)$ $= 90°$	[sum of int ∠'s of Δ]	✓ S ✓ S
	∴ KM = ML	[line from centre ⊥ to chord]	✓ R (3)
	OR/*OF*		
	$O\hat{K}T = 54°$	[∠s opposite = radii]	
	$\hat{K}_1 = 54° - 18° = 36°$		✓ S
	$T\hat{M}K = 90°$	[sum of int ∠'s of Δ]	✓ S
	∴ KM = ML	[line from centre ⊥ to chord]	✓ R (3)

8.2

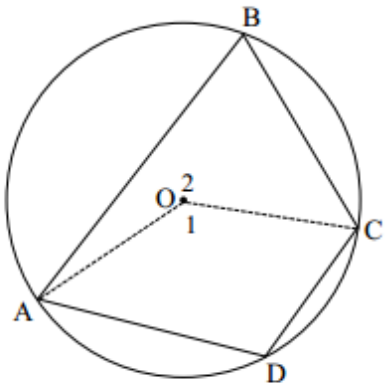


8.2.1	$\frac{DC}{CS}=\frac{20}{12}=\frac{5}{3}$ $\therefore \frac{DC}{CS}=\frac{RB}{BS}$ $\therefore$ BC ‖ DR [converse line ‖ one side of Δ OR sides in the same proportion] $\therefore$ BC ‖ AD	✓ S ✓ S ✓ R (3)
8.2.2	$\frac{AR}{AD}=\frac{RB}{BS}$ [line ‖ one side of Δ] **OR**[Prop Theorem AB ‖ DS] $\frac{AR}{AD}=\frac{5}{3}$ $\frac{48-AD}{AD}=\frac{5}{3}$ $\therefore 5AD = 144-3AD$ $AD = 18$ AB = 20 [opp sides of parm] $\therefore$ AD : AB = 18 : 20 = 9 : 10	✓ $\frac{AR}{AD}=\frac{5}{3}$ ✓ AD = 18 ✓ ratio (3)

	OR/OF $\frac{AR}{RD}=\frac{5}{8}$.....prop thm AB || DS $\frac{AR}{48}=\frac{5}{8}$ $\therefore AR=30$ and $AD=18$ $\therefore \frac{AR}{RD}=\frac{AB}{DS}$.....$\|\|\|\ \Delta's$ $\therefore AB=20$ $\therefore AB:AD=18:20=9:10$	✓ $\frac{AR}{RD}=\frac{5}{8}$ ✓ $AD=18$ ✓ ratio (3)
		[15]

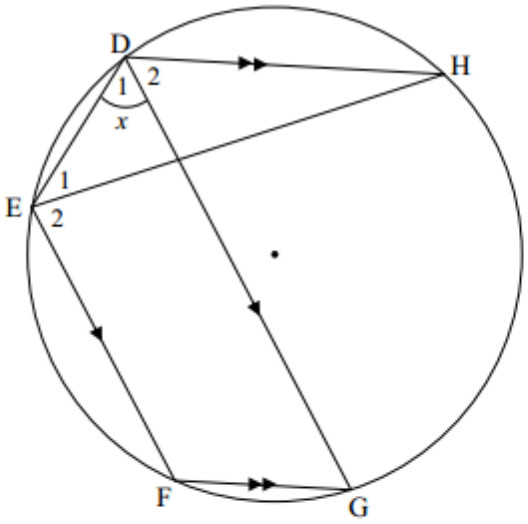
QUESTION/*VRAAG 9*

9.1



9.1	Constr: Draw radii OA and OC. Proof: $\hat{O}_1=2\hat{B}$ [∠ at centre = 2×∠ at circumference] $\hat{O}_2=2\hat{D}$ [∠ at centre = 2×∠ at circumference] $\hat{O}_1+\hat{O}_2=360°$ [revolution] $2\hat{B}+2\hat{D}=360°$ [revolution] $\therefore \hat{B}+\hat{D}=180°$	✓ Construction ✓ S ✓ R ✓ S/R ✓ S (5)

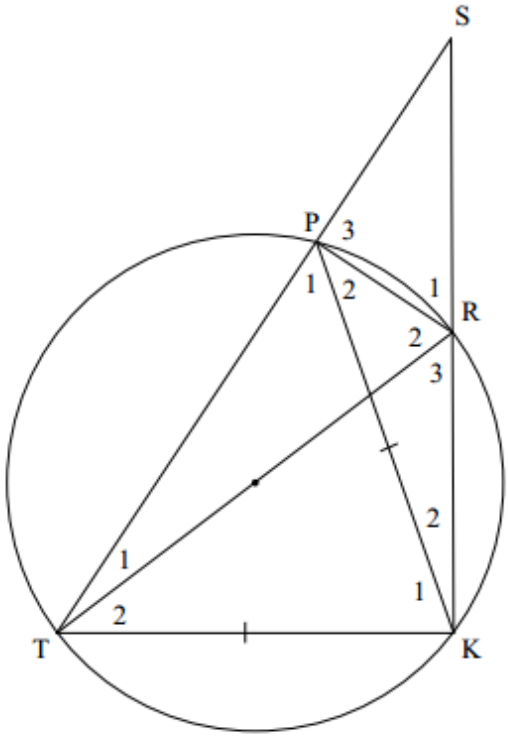
9.2



9.2	$E\hat{F}G = 180° - \hat{D}_1$ [opp ∠'s of cyclic quad] $\therefore E\hat{F}G = 180° - x$ $E\hat{F}G = 180° - \hat{G}$ [co-int ∠'s; EF ‖ DG] $\hat{G} = x$ But $\hat{G} = \hat{D}_2$ [alt ∠'s; DH ‖ FG] $\therefore \hat{D}_1 = \hat{D}_2 = x$	✓S ✓ R ✓S / R ✓ S / R (4)
		[9]

QUESTION/VRAAG 10

10.1



10.1.1	$T\hat{P}R = 90°$	[∠ in semi-circle]	✓S ✓R
	$S\hat{P}R = 90°$	[∠'s on a straight line]	✓S
	∴ SR is a diameter	[converse ∠ in semi-circle]	✓R
			(4)
	OR		
	$T\hat{K}R = 90°$	[∠ in semi-circle]	✓S ✓R
	$S\hat{P}R = 90°$	[ext ∠ of cyclic quad]	✓S
	∴ SR is a diameter	[converse ∠ in semi-circle]	✓R
		OR	(4)
		[chord subtends a right angle]	

10.1.2	$\hat{R}_1 = P\hat{T}K$ [ext ∠ of cyclic quad] $\hat{P}_1 = P\hat{T}K = \hat{R}_1$ [∠s opp equal sides] $\hat{S} + \hat{R}_1 = \hat{P}_1 + P_2$ [ext ∠ of Δ] $\therefore \hat{S} = \hat{P}_2$ [$\hat{R}_1 = \hat{P}_1$]	✓S ✓R ✓S /R ✓S ✓R (5)
10.1.3	In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] ΔSPK ||| ΔPRK [∠, ∠, ∠] **OR/*OF*** In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] $S\hat{P}K = P\hat{R}K$ [sum of ∠s in Δ] ΔSPK ||| ΔPRK	✓S ✓S ✓S/R (3) ✓S ✓S ✓S/R (3)
10.2	$\frac{PK}{RK} = \frac{SK}{PK}$ [ΔSPK ||| ΔPRK] $PK^2 = SK.RK$ $ST^2 = SK^2 + TK^2$ [Pythagoras] $TK = PK$ [Given] $ST^2 = SK^2 + PK^2$ $ST^2 = SK^2 + SK.RK$ $ST^2 = (2RK)^2 + 2RK.RK$ $ST^2 = 6RK^2$ $ST = \sqrt{6}RK$	✓S ✓S ✓ $PK^2 = SK.RK$ ✓SK = 2RK ✓ $ST^2 = 6RK^2$ (5)
		[17]

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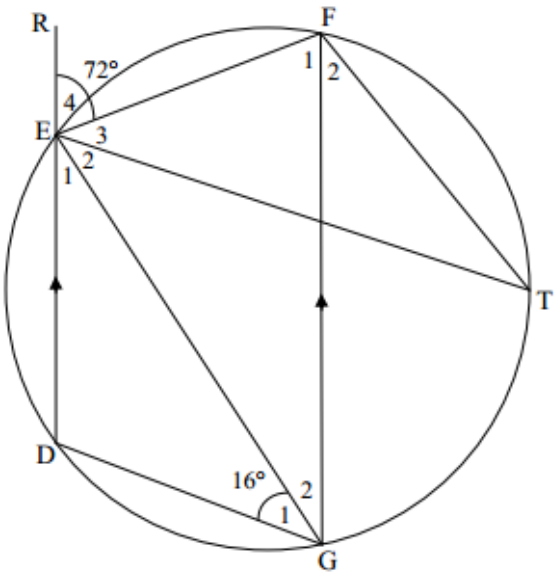
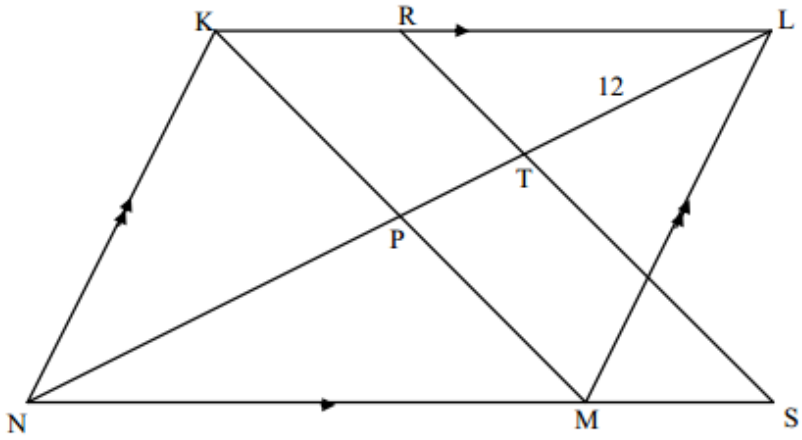
QUESTION/*VRAAG* 9

9.1



9.1.1	$D\hat{G}F = \hat{E}_4 = 72°$ [ext ∠ of cyclic quad/ *buite ∠ v kvh*]	✓ S ✓ R (2)
9.1.2	$\hat{G}_2 = 72° - 16° = 56°$ $\hat{T} = \hat{G}_2 = 56°$ [∠s in the same seg/ *∠e in dies.* ⊙ *segment*]	✓ S ✓ S / R (2)
9.1.3	$\hat{F}_1 = \hat{E}_4 = 72°$ [alt ∠s; DE ∥ GF / *verw. ∠e*; DE ∥ GF] $\therefore G\hat{E}F = 52°$ [sum of ∠s in Δ / *∠e van* Δ] **OR/*OF*** $\hat{E}_1 = 56°$ [alt ∠s; DE ∥ GF / *verw. ∠e*; DE ∥ GF] $\therefore G\hat{E}F = 52°$ [∠s on a str. line/ *∠e op 'n reguitlyn*]	✓ S / R ✓ S (2) ✓ S / R ✓ S (2)

9.2

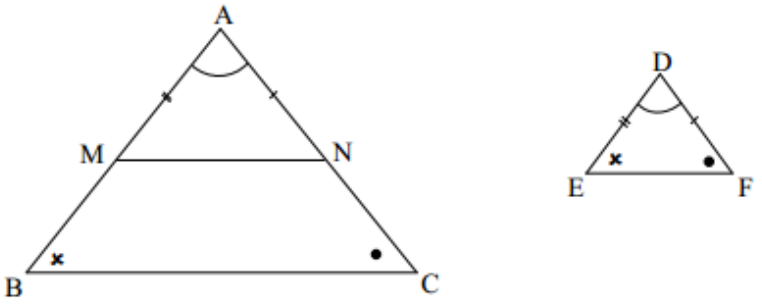


9.2.1	NP = PL = 16 [diag of ‖m / *hoeklyne van ‖m*] PT = 4 NP : PT = 16 : 4 = 4 : 1	✓ S ✓ R ✓ S ✓ answer (4)
9.2.2	NM : MS = 4 : 1 NP : PT = NM : MS KM ‖ RS [line divides two sides of Δ in prop / *Lyn verdeel 2 sye v Δ eweredig*] **OR/*OF*** [converse prop theorem / *omgekeerde lyn ‖ een sy v Δ*]	✓ S ✓ R (2)
9.2.3	$\frac{RL}{KL} = \frac{TL}{LP}$ [prop theorem; KM ‖ RS **OR** line ‖ one side of Δ / *Lyn ‖ een sy v Δ*] $RL = \frac{12 \times 21}{16}$ = 15,75	✓ S ✓ R ✓ S ✓ answer (4)

	OR / *OF*	
	NM : MS = 4 : 1	
	KR = MS = 5,25 [opp side of $\|\|^{m}$ / *teenoorst. sye van* $\|\|^{m}$]	✓ S ✓ R
	KL = NM = 21	
	RL + 5,25 = 21	✓ S
	RL = 15,75	✓ answer (4)
		[16]

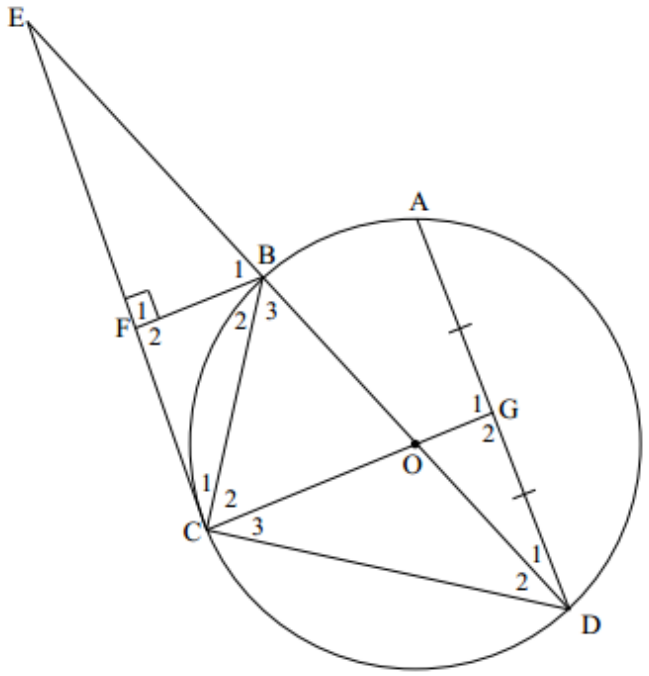
QUESTION/*VRAAG* 10

10.1



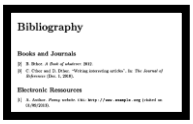
10.1	Constr: Let M and N lie on AB and AC respectively such that AM = DE and AN = DF. Draw MN.	✓Constr
	Proof: In Δ AMN and Δ DEF	
	AM = DE [Constr / *Konstruksie*]	
	AN = DF [Constr / *Konstruksie*]	
	$\hat{A} = \hat{D}$ [Given /*Gegee*]	
	$\therefore$ Δ AMN ≡ Δ DEF [*s*, ∠, *s*]	✓S ✓R
	$\therefore A\hat{M}N = \hat{E} = \hat{B}$	
	MN || BC [corresp ∠'s are equal/ *ooreenk. ∠e gelyk*]	✓S /R
	$\frac{AB}{AM} = \frac{AC}{AN}$ [line || one side of Δ **OR/*OF*** prop theorem; MN ||BC / *Lyn* || *een sy v* Δ]	✓S ✓R
	$\therefore \frac{AB}{DE} = \frac{AC}{DF}$ [AM=DE and AN=DF]	(6)

10.2



10.2.1(a)	$F\hat{C}O = 90°$ [tan ⊥ radius / *raaklyn ⊥ radius*] $\hat{F}_1 = 90°$ [BF ⊥ EC] $\therefore F\hat{C}O = \hat{F}_1 = 90°$ FB ‖ CG [corresp ∠s = / *ooreenk. ∠ gelyk*]	✓ S / R ✓ S ✓ R (3)
10.2.1(b)	In ΔFCB and ΔCDB $B\hat{C}D = 90°$ [∠ in semi-circle / $\angle \frac{1}{2}\odot$] $\hat{F}_2 = 90°$ [BF ⊥ EC] $\therefore \hat{F}_2 = B\hat{C}D = 90°$ $\hat{C}_1 = \hat{D}_2$ [tan chord theorem / *∠ tussen rkl en koord*] $\hat{B}_2 = \hat{B}_3$ [sum of ∠s in Δ / *∠e van Δ*] $\therefore$ ΔFCB ||| ΔCDB	 ✓ S /R ✓ S ✓ S ✓ R ✓ S
	OR/*OF* In ΔFCB and ΔCDB $B\hat{C}D = 90°$ [∠ in semi-circle / $\angle \frac{1}{2}\odot$] $\hat{F}_2 = 90°$ [BF ⊥ EC] $\therefore \hat{F}_2 = B\hat{C}D = 90°$ $\hat{C}_1 = \hat{D}_2$ [tan chord theorem / *∠ tussen rkl en koord*] $\therefore$ ΔFCB ||| ΔCDB [∠,∠,∠]	 ✓ S / R ✓ S ✓ S ✓ R ✓ R (5)

10.2.2	$\hat{G}_1 = 90°$ [line from centre to midpt of chord / *midpt. ⊙; midpt. koord*]	✓ R (1)
10.2.3	In ΔGCD and ΔCDB $\hat{G}_2 = B\hat{C}D = 90°$ $\hat{C}_3 = \hat{D}_2$ [∠s opp equal sides /∠*e teenoor gelyke sye*] $G\hat{D}C = \hat{B}_3$ [sum of ∠s in Δ /∠*e van* Δ] $\therefore$ ΔGCD ||| ΔCDB [∠,∠,∠] $\therefore \frac{CD}{DB} = \frac{CG}{CD}$ [||| Δs] $\therefore CD^2 = CG.DB$	✓ identifying Δs ✓ S ✓ S / R ✓ S **OR** ✓ R ✓ S (5)
10.2.4	$\frac{BC}{DB} = \frac{FB}{BC}$ [ΔFCB ||| ΔCDB] $\therefore BC^2 = DB.FB$ $CD^2 + BC^2 = CG.DB + DB.FB$ $DB^2 = DB(CG + FB)$ $DB = CG + FB$	✓ S ✓ R ✓ S ✓ sum ✓ $DB^2 = CD^2 + BC^2$ (5)
		[25]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